Systematic and Metasystematic Reasoning: A Case for Levels of Reasoning beyond Piaget's Stage of Formal Operations

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COMMONS, MICHAEL L., RICHARDS, FRANCIS A., and KUHN, DEANNA *Systematic and Metasystematic Reasoning A Case for Levels of Reasoning beyond Piaget's Stage of Formal Operations* CHILD DEVELOPMENT, 1982, **53**, 1058–1069 Modes of cognition are postulated consisting of third- and fourth-order operations, they are hypothesized to be qualitatively distinct from, and hierarchically related to, the form of reasoning characterized as formal operational by Inhelder and Piaget An instrument was developed to assess these modes of cognition, labeled systematic and metasystematic reasoning, and was administered to 110 undergraduate and graduate students The results support the assertion that systematic and metasystematic reasoning exist as modes of cognition discrete from, and more complex and powerful than, formal operational reasoning

The purpose of the present paper is to explore the possibility that modes of systematic reasoning can be identified that are qualitatively distinct from, and logically more complex than, the form of reasoning characterized by Inhelder and Piaget (1958) as "formal operational" In this endeavor, we have made use of Piaget's system of successive stages of logical operations only in its most general form In other words, Piaget's stages are regarded as successive levels of cognitive representation, in this sense, the stages are not subject to any form of empirical verification, but rather constitute a conceptual description of forms of cognition (Bickhard 1979) Thus, sensory-motor operations are defined as actions on material objects, symbolic operations as actions or operations on symbolic representations of material objects (e g , the concrete operational organization of symbolically represented objects into classes or relations), and formal operations as operations on the above-mentioned operations (of classification or relation), for example, the individual organizes classes (of elements) according to relations that obtain among them The object of the present work was to inquire whether levels of higher-order operations could be identified and, if so, to establish the particular form they would take in the cognitive performance of the individual

There are several existing attempts in the developmental literature to postulate "postformal operational" stages of cognition, notably those of Riegel (1973) and Arlin (1975) The major problem with these formulations is that it is uncertain the extent to which these various postulated modes of cognition constitute a qualitatively distinct structure or stage of reasoning, related in a hierarchical manner to formal operations, or, alternatively, whether they are modes of cognition that develop parallel to Piaget's stage sequence Forms of Arlin's stage of "problem finding," for example, are likely to be present during the stages of concrete and formal operations (Fakouri 1976), thus, the stage of formal operations cannot be regarded as the necessary but insufficient condition for the stage of problem finding, as it would need to be if the two were to be related in a hierarchical manner A similar consideration applies to Riegel's "dialectic operations" As Riegel (1973) noted, dialectical operations are potentially present in some form at all of Piaget's stages

It is our view, then, that if forms of advanced cognition are to be identified that make

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reference to, and build on, Piaget's sequence of stages of cognition, they must take the general form of the higher-order operations on operations on operations (and possibly beyond) referred to above To make this claim is not to assert that all forms of adult cognition necessarily fall within such categories Indeed, current research and theory in adult cognitive development run strongly counter to such a claim (Kuhn, Pennington, & Leadbeater, in press, Labouvie-Vief 1980, 1982) Rather, our claim is that if one wishes to postulate more advanced forms of logical/mathematical reasoning of the type studied by Piaget, that build on the forms of reasoning that constitute his existing stage sequence, then this more advanced reasoning must take the general form we have indicated

What might such higher-order operations look like in the cognition of an individual? We can refer to the second-order, or formal, operations in Piaget's system as reflecting "interrelational thinking " That is, reasoning is based on ordered relations of classes or relations, the individual executes operations on these classes or relations For example, the individual may formulate and test hypotheses about the relations that obtain among two classes of objects In executing these formal operations, however, the individual does not self-consciously represent, reflect, or operate on the system as a whole We postulate, then, a possible set of third-order operations, termed "systematic operations," which consist of exhaustive operations on classes or relations of classes or relations, forming systems We further postulate the possibility of a set of fourth-order operations, termed "metasystematic operations," which consist of operations on systems

Systematic operations apply to the entire set of constituents of a system (that itself is made up of operations on operations) For example, they might consist of the coordination of iterative operations such as Σ_i or the coordination of abstract representations of operations, such as $a \circ b$, $a * b$, to form systems Such systematic descriptions formally represent the properties of operations at the formal operational level

Metasystematic operations are cognitions about systems They are required in the formation of a framework (or "metasystem") for comparing and contrasting systems with one another The relationship of one system to another such system is expressed as a metatheory and is found by comparing axioms, theorems, or other limiting conditions of systems within the framework of a "super-system" that contains all of the variant systems Metasystematic reasoning is defined as the set of operations necessary to construct the supersystem and to execute the analysis of the systems contained therein

An example of metasystematic reasoning is found in that aspect of Einstein's general theory of relativity that deals with the coordination of inertial and gravitational mass Prior to Einstein's formulation, there was one system for describing inertial mass and a separate one for describing gravitational mass Each separate system included *systematic* representations of properties of *formally* defined relations between variables Inertial mass was the property of the system that described a body's resistance to acceleration Gravitational mass was a property of another system that described the weight of a body in a given gravitational field In fact, since the same constant for mass represents both the inertia and the weight of a body, it follows that it is impossible to discover by experiment which of the following is true The motion of a given system of coordinates is straight and uniform and the observed effects within the system are due to a gravitational field, or the system of coordinates is uniformly accelerated and the observed effects within the system are due to inertia Recognition of an equivalence between the two cases constitutes the equivalence principle of general relativity theory This principle states that the inertial system is isomorphic (contains the same structure and elements) to the gravitational system, that is, any relationship that is true in one is true in the other Whether the same property appears as weight or as inertia depends on which description of the coordinate systems is employed, that is, motion in a gravitational field appears in relation to an inertial system of coordinates, while motion in the absence of a gravitational field appears in a coordinate system that is accelerated This principle requires a coordination of two distinct systems, each generated by systematic operations the inertial system and the gravitational system The cognitive operations that effect this coordination are at a "metasystematic" level, distinct from that of the operations applied within either of the individual systems

The above example serves to illustrate the irreducibility of the higher-order operations to operations of the next lower order Operations at the metasystematic level must be expressed in the language of metalogic, or its psychological equivalent, because statements about the relation between systems cannot be reduced to statements about the properties of the relations within any single system Similarly, statements about systems properties cannot be contained within any of the individual systems Hence, systematic operations cannot be reduced to formal operations because the formal operational systems themselves do not contain descriptions of the system These formal operations are not rich enough to describe their own properties If this were possible, Russell's paradox (the sentence, "This sentence is false") would not exist In other words, a property of a system cannot be described by a proposition within a system

Method

Task

In the problem we developed to assess systematic and metasystematic reasoning, the subject was asked to compare and contrast four systems, each comprised of a set of asymmetric relations Two similar forms of the problems were constructed One form of the problem and the instructions given to the subject are shown below

Here are four stories After you read them, you will answer questions on which are most similar and which most different Use the "greater than" symbol "$>$" to indicate the order of things For example, indicate "Brown prefers Oregon over Texas, by $O > T$ Only attend to order

1) On counter earth, Richard Reagan has been elected President of the United States As a gesture of gratitude to the people of his home state, California, Reagan has convinced them to leave California and either wander around with no state, or take a combination of one, two, or three of the following states Oregon, Washington, and Indiana Reagan thinks that the economic value of Oregon and Washington are equal, and that the value of either is less than that of Indiana The boundary and passport bill which he submitted to Congress, requiring the barbed wiring and mining of the boundaries at considerable expense, did not pass Instead, Congress specified that the funds asked for in that bill were to go to the states selected to be used as the states saw fit Only Washington and Oregon have common boundaries and, therefore, a union of these two states would receive a smaller amount of the fortification funds Therefore, even with the value of their combined economies, Reagan thinks that the pair Oregon and Washington is less valuable than the pairs Oregon and Indiana or Washington and Indiana Since Indiana makes boats which can be used on the Columbia River, Reagan thinks that the economic benefits of pairing Washington and Indiana are slightly greater than the benefits of pairing Oregon and Indiana Reagan thinks that three states are worth more than any combination of 0, 1, or 2 states, and any 2 states are worth more than 0 or 1 states, except he cannot decide which is worth more, Indiana or the pair Oregon and Washington

2) Bad Bart ambles into the local casino and converts his gold watch into chips of the following colors silver, bronze, and gold Bart likes to play the one-chip candy machines He likes the chips in the following order gold better than silver, silver better than bronze, and gold better than bronze, and any chip over none Bart also likes to play the one-armed bandit machines which use combinations of two chips He likes the two-chip combinations in the following order golds and silvers first, golds and bronzes second, and silvers and bronzes third With one exception, Bart knows he likes to play with any two chips over one or none, he is not sure about a gold versus a silver and a bronze Because Bad Bart likes to play with three chips best, there is one machine that he likes better than any other the washing machine, and it takes a silver, a bronze, and a gold chip

3) In Madras, India, V P Vanktesh, a man of habit and variable income, has a favorite restaurant Although his tastes never vary, the food he can afford does Of the three foods the restaurant serves he prefers curry, birani, and alu paratha, in that order Also, he likes curry better than alu paratha, and anything better than nothing When V P has more money he buys two dishes, except it is not known whether he would choose the curry over both the birani and the alu paratha He likes the combination of curry and birani better than curry and alu paratha, and he also likes curry and alu paratha better than birani and alu paratha, and birani and alu paratha better than curry and birani Although a temperate man, at festivals, given the means, he has all three dishes instead of any single dish or pair

4) A jeweler has three boxes, the first containing different kinds of broken 18-carat gold necklaces, the second various scratched earrings, and the third different kinds of 18-carat gold pins that are broken He keeps the old jewelry because he occasionally uses the gold To get the approximate amount of gold he needs, he weighs and then melts down a combination of objects To do the weighing he uses a simple balance-beam scale It consists of a beam that pivots in the middle and two pans hanging from each end of the beam, equidistant from the pivot The beam is level when the pans are empty The jeweler can place combinations of from 0 to 3 object types into one pan, but never more than one of each object type in a pan Begin-

ning with empty pans, he notices that whenever he puts any combination containing at least one object into a pan, that pan sinks down, indicating that it is heavier than the empty pan Using this same method, he finds that any pin is heavier than any earring Necklaces are always heavier than pins He has discovered two rules that reduce how many combinations he needs to try to find out how the weights of the combinations are ordered First, he notices that if the combination in the right pan is heavier than the combination in the left pan, and a single object type not already in either of the pans is added to both pans, the right pan remains heavier than the left Secondly, if he weighs three combinations of objects, he finds that the following is always true If the first combination is heavier than the second, and the second heavier than the third, then the first is heavier than the third

Space was left after each story for subjects to make notations On a separate page, the following instructions were presented

Now that you have read the stories and are familiar with them, answer the questions below Make your comparisons on the basis of properties of the orderings found in each story, in the best and most complete way that you can Write out all the comparisons that you can in a systematic way Use symbols to represent the order of things, and explain what the symbols stand for Also include an explanation in English You may want to use a chain, for example, $\begin{array}{c}\text{Oregon}\\ \downarrow\\ \text{Texas}\end{array}$, in addition to an ordering, Oregon > Texas Then explain what are the most important similarities and differences in the stories, and explain how you arrived at deciding the relative importance of these similarities and differences You may refer to more than one framework Make sure to give the strongest, most thorough, broad, inclusive, and complete explanations possible for the similarities and differences It is necessary that you show all of your work in forming the orders and making the comparisons, as well as your commentary in English

The following form was used for subjects to provide their answers

1 Which of the stories are the most similar?

1 and 2___	2 and 3___	1, 2, and 3___	1, 3, and 4___
1 and 3___	2 and 4___	1, 2, and 4___	1, 2, 3, and 4___
1 and 4___	3 and 4___	2, 3, and 4___	

2 Which of the stories differ the most from the ones you listed as similar in (1), even though they may have many characteristics in common?

1___	4___	1 and 2___	2 and 3___
2___		1 and 3___	2 and 4___
3___		1 and 4___	3 and 4___

3 Explain why the stories you listed above as similar are similar

4 Explain why the stories you listed as different from these are different

Space was provided for subject's answer [1]

Each of the four systems within the problem consists of a finite, partially ordered, commutative semigroup, with a binary operation for combining objects and a partial order relation defined among all possible combinations of elements The systems were presented as stories about which combinations of three objects, a, b, c, were greater than others The combinations could include no item, single items, or two or three items In the first three stories, for the most part, the orderings of combinations of objects were explicitly stated One object is preferred over another, or over none at all, for example, a over none, c over b, b over a, and so forth, some pairs are preferred over others, for example, $b + c$ over $a + b$, or the complete group is preferred over a pair, for example, $a + b + c$ over $b + c$ In the fourth story, the structure of the order was stated in propositional form which did not provide direct access to the orderings of the elements, these had to be derived by the subject

An alternate form B was presented to a portion of the sample Forms A and B were identical except for the structure reflected in story 1 (see fig 1) and the names of the elements in each of the stories, for example, the names of the states in story 1 and the foods in story 3

The subject was allowed an unlimited amount of time The average amount of time spent was 1 hour, with an approximate range from 30 min to 2 hours

Two additional problems were presented to a portion of the subjects (73 of the 110) a simple transitivity problem (requiring concrete operations) and a version of Inhelder and Piaget's (1958) pendulum problem (requiring formal operations) Problem order was counterbalanced It was hypothesized that performance on the three problems would show a hierarchical pattern, that is, no subject would master the pendulum problem who did not master the transitivity problem, and no subject would master

[1] This problem is used with the permission of the Dare Association, Inc

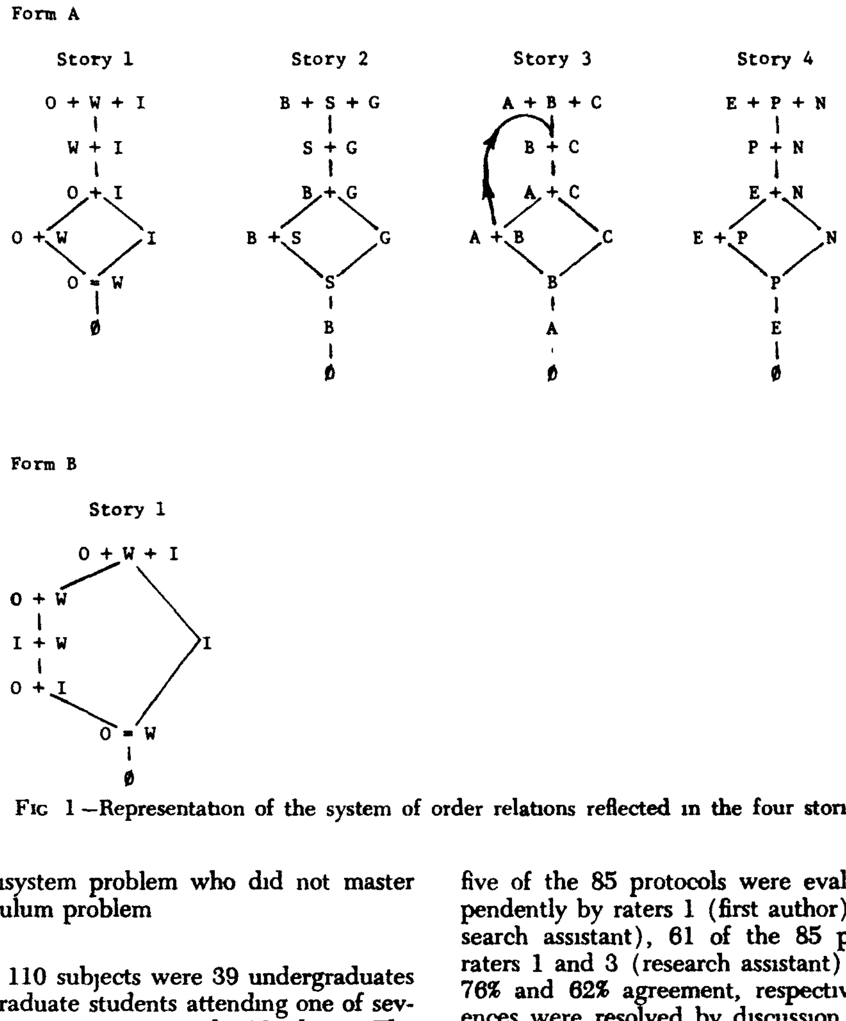

FIG 1 —Representation of the system of order relations reflected in the four stories

the multisystem problem who did not master the pendulum problem

Subjects

The 110 subjects were 39 undergraduates and 71 graduate students attending one of several private universities in the Northeast The mean ages for the groups are 20 6 and 26 1 years, respectively All participated on a volunteer basis The multisystem problem was administered a second time to 41 of the 71 graduate students, directly following the initial administration for 22 of them, and 8 months following the initial administration for 19 of them (Half of the students received form *A* first, the other half form *B* first)

Results

Analysis of subjects' protocols suggested six distinct levels of response. The scoring system was developed based on an intensive analysis of 25 of the protocols, guided by the theoretical perspective set forth in the introduction It was then applied to the remaining 85 Fifty-five of the 85 protocols were evaluated independently by raters 1 (first author) and 2 (research assistant), 61 of the 85 protocols by raters 1 and 3 (research assistant) There was 76% and 62% agreement, respectively Differences were resolved by discussion and a final level assigned to each protocol

Level C —The initial level, labeled *C*, resembles the mode of thinking termed concrete operational in Piaget's system Subjects categorized in this level base their judgments of similarity/dissimilarity on superficial features of the stories rather than on order relations (either within or across stories), as the task instructions direct If order relations are attended to at all, this attention is limited to a representation of the discrete order relations explicitly given in the story The subject does not perform any operations on these elements, to derive additional order relations or more general properties of the system (story) Subject RA provides an example (The examples to follow are based on both form *A* and form *B*, so that letters may not always match the stories given earlier)

Under story 1, RA wrote

$U = C$, $U < I$, $C < I$, $CI > UI$,
 ($I \underset{?}{\geq} UC$), $3 > 2 > 1$

Under story 2, RA wrote

He likes $B > R$, $R > W$, $B > W$,
 ($3 > 2 > 1$), $BR > BW > RW$,
 ($B > RW$)?

Under story 3, RA wrote

He prefers $CK > Bak > Yog$,
 $Ck - Ba > Ch - Yo$,
 $Ch - Yo > Ba - Yo$,
 $Ba - Yo > Ck - Ba$?,
 ($3 > 2 > 1$) ($Ch > Ba - Yo$)?

Under story 4, RA wrote

$Br > Ri$, $Wa > Br$, ($Wa > Ri$),
 ($Wa > Br - Ri$),
 ($3 > 2 > 1$) comb, right pan, left pan

RA then wrote "Stories 1, 2, and 3 are most similar because in the first three stories a person's opinion is involved He thinks, or he likes, or he prefers, a particular object or a combination of objects, and if another person were to make a choice on what he would prefer of such combinations, the combinations would be likely to change (e g , some people may like white chips better than blue) At the same time, story 4 is based on facts, and if the jeweler were replaced, the facts about the combinations would still be true " Note that RA does not link information from discrete order relations into a higher-order unit or chain (though we know RA is capable of a transitivity inference) $B > R$, $R > W$, and $B > W$ are not integrated into a unified order relation $B > R > W$

Level F—This level is postulated to be equivalent to the level of formal operations in Piaget's sequence (Piaget's levels IIIA & B, see Inhelder & Piaget [1958]) Order relations within stories are operated on in a systematic manner, but the entire system is not regarded (operated on) as a single entity having characteristic properties that may be compared with the properties of other systems Thus, the subject's attempt to relate the stories with respect to their similarity/dissimilarity is limited to establishing that single elements or two-element order relations map to some degree from one of the first three stories to another Subject LR provides an example Under story 1, LR wrote

$I > W$, $O, W > W + I > OI$,
 $3 > 2$ $O + W + I$, uncertainty $I ? O, W$

Under story 2, LR wrote

$g > s > b >$ none, one exception
 $gs > gb > sb$, one uncertainty $g ? b, s$

Under story 3, LR wrote

$c > b > a >$ none,
 uncertainty in one case $b, a ? c$

Under story 4, LR wrote

$n > p > e >$ none,
 no uncertainty, no pairing

LR then wrote "1 and 3 are similar because both individuals prefer any 3 to any 2, any 2 to any 1, and any 1 to any none, with one uncertainty In story 1, Reagan does not know if $I > O$, W, and in story 3, Vanktesh does not know if B, $A > C$ One difference is that the two singles are equal to each other in story 1 ($W = O$), whereas this is not true of story 3 ($C > B > A$) The other difference is that the Californians may or may not benefit most from taking $I + O$ over W, whereas Vanktesh definitely would take all three "

Level S—Responses at this level reflect the application of what we have termed systematic reasoning At this level, subjects clearly show that they understand that the logical structure of each story must be examined as an integral whole or structure In representing the structure of each story, the subject may choose one of two possible courses A schematic representation of each of the systems can be generated and these representations compared with respect to their deviations from one another Alternatively, the subject can represent the systems on the basis of the axioms that do or do not characterize each of the systems The two methods yield equivalent results Use of these representations is taken as evidence that the subject perceives the story as a system, that is, a coherent whole which determines the internal pattern of relations across its elements When systematic operations are fully consolidated, full representations are constructed There is no evidence for the presence of a framework for intersystem comparison, cognition is focused on intrasystem analysis Subject

NJ provides an example Under story 1, NJ wrote

$$O = W, O < I, W < I,\ \ O + W > O + I,$$
$$O + W > W + I,\ \ W + I > O + I,$$
$$O + W + I > W + I,$$
$$O + W + I > O + I,$$
$$O + W + I > W + O,\ \ O + I > W,$$
$$O + W \overset{?}{>} I,\ \ I + W > O$$

Under story 2, NJ wrote

$$G > S,\ \ S > B,\ \ G > B,\ \ G > O,$$
$$B > O,\ \ S > O,\ \ G + S > G + B >$$
$$S + B,\ \ G \overset{?}{=} B + S,\ \ G + B > S,$$
$$G + S > B,\ \ G + S + B > G + S >$$
$$G + B > S + B$$

Under story 3, NJ wrote

$$C > B > A,\ \ C > A,\ \ C > O,\ \ A > O,$$
$$B > O,\ \ C \overset{?}{=} B + A,\ \ C + B > A,$$
$$C + A > B + A,\ \ B + A > C + B,$$
$$A + B + C > B + A,\ \ A + C,$$
$$B + C,\ \ A,\ \ B,\ \ C$$

Under story 4, NJ wrote

$$N > O,\ \ E > O,\ \ P > O,\ \ P > E,$$
$$N > P$$

(Note that story 4 is incompletely represented) NJ then wrote "1 and 2 both use the technique of comparing items in regard to their relative merit on a one-to-one basis, then in pairs In addition, the relative worth of combination versus single item and versus zero is analyzed. 1 and 2 also follow the transitive law of geometry, that is $x > y$ and $y > z$, then $x > z$ The law holds for 1 and 2, whereas it fails in example 3 In 3, $B + A > C + B$, then by taking away B, $A > C$, but this contradicts what we are told before I realize, however, that Mr Vanktesh's preferences need not follow mathematical laws and logic, he may indeed prefer the $B + A$ combination better I point this out because I felt this made 3 less similar to 1 than 2 was to 1 " NJ's reasoning is characterized by an effort to describe differences across stories Stories are compared in a pairwise fashion in this effort, evidence for the lack of a systematic framework for comparison

*Level M*1 —At this level the first evidence of metasystematic operations appears Comparisons across stories are based on variations of the structural properties, implicitly indicating the possibility of conceptualizing one story as the transformation of another story At level $M1$, the subject generates a much more complete set of lattices and/or axioms to represent or characterize the stories However, some axioms or orderings necessary to complete the analysis are clearly missing Subject CC serves as an example by providing only a partial analysis of story 4 (leaving out the details of its ordering and leaving out the single combinations of the elements)

Subject CC's analysis is as follows

Under story 1 CC wrote

$I > O = W,\ \ O + W + I > O + W > \underbrace{W + I > O + I} > I > O = W$ where do these fit?

$O + W >$ fortify $> O + I,\ \ W + I,$

$W + I > O + I,\ \ I \overset{?}{=} O + W$

Under story 2 CC wrote

$G > S > B >$ none, $\ GSB > GS > GB > SB \overset{?}{=} G > S > B >$ none, $\ GS > GB > SB$

Under story 3 CC wrote

$C > B > A >$ nothing, $\qquad C + B > C + A,$

$C > A >$ none, $\qquad C + A > B + A,$

$C \overset{?}{>} B + A, \qquad B + A > C + B,$

$C + B + A > C + B > C + A > B + A \overset{?}{>} C > B > A >$ none

$C + B + A > C + B > C + A > B + A > C + B$

circular

Under story 4 CC wrote

$N > P > E,\ \ R > L,\ \ NP > NE,\ \ NP > NE > PE,\ \ PR\ PE,\ \ VP > PE,\ \ PN\ PE,$
$NE > PE$

CC then wrote "[In stories 1 and 3], there is a basic similarity in the structure combinations of three, combinations of two, one, none However, in both stories, there is a degree of uncertainty as to precise order of the combinations In story 1, $O + W + I > O + W > W + I > O + I > I > O = W$, and also the ranking of I versus $O + W$ is uncertain Therefore, if I is greater than $O + W$, then the preferences become 'circular' as in story 3 If $I = O + W$, then again the preferences are circular $O + W = I > W + I > O + I > I$ In story 3, the circular nature of the combinations is spelled out $C + B + A > C + B > C + A > B + A > C + B$ and $B + A > C > B > A >$ none Story 4 differs the most Both 2 and 4 have very clear relationships Though the relationships in story 4 are not explicitly spelled out, there is enough information to deduce that a combination of 3 > combinations of 2 > singles In story 2, the relationship is also the same, though there is slight uncertainty as to the relationship between SB and G Nonetheless, this uncertainty is not enough to change the order of the relationships the way it does in stories 1 and 3"

It is clear that CC's reasoning and representations could be carried further On the other hand, CC does use "circularity" and uncertainty of order to refer to the integrated stories as wholes, and she generates lattices for three stories Structure is compared across stories, even though incorrectly The use of circularity and "uncertainty" indicates the existence of a common "source structure" whose properties are altered to produce the different instantiations of structure found in the stories

Level M2 —This is the second level in the emergence of truly metasystematic operations This level, and the two which follow it, are progressive steps in the consolidation and organization into a whole, of operations involved in comparisons across systems Subjects at this level have, either explicitly or implicitly, full and integrated representations of the systems of ordered relations reflected in each of the four stories (Systematic thinking is now fully consolidated) These representations are used to check (again either explicitly or implicitly) their assertions about the systems in a systematic and complete fashion These subjects choose a single property which is appropriate for comparing the integrated structures of the stories, as opposed to one that would enable comparisons of parts only For example, the subject might compare the stories on the basis of either additivity or transitivity properties Such properties are used to construct comparison frameworks For a subject who represents the systems underlying the stories graphically, the comparison framework may consist simply of a set of dimensions along which the physical drawings of the systems are tested for resemblance A subject may imply that he or she ~~has~~ generated complete orderings ~~in all~~ four stories without actually showing the work Thus, the simple assertion that "stories 2 and 4 follow the law of additivity while 1 and 3 do not," is enough to classify a subject at level $M2$ (In protocols classified at levels $M1$ or S, in contrast, there exists clear evidence that certain combinations, i e , relations between elements, were not considered in generating the representation) Subject BR provides an example of level $M2$

Subject BR represented the stories as follows

Under story 1, BR wrote

$O = W$ common boundaries economically

$O < I$, $I > O$, $O + W >$ fortifying, $W + I > O + I$,

$W < I$, $I > W$, $O + I <$ fortifying, $3 >$ pair,

$W + I <$ fortifying, pair $>$ single

except $I \overset{?}{} O + W$

Under story 2, BR wrote

$G > S$, $\underbrace{G + S > G + B > S + B}$, 3 chips $>$ 2 chips,

$S > B$, better than 1 chip,

$G > B$ except $G \overset{?}{=} S + B$

Under story 3, BR wrote confused Indian or exam taker'

$C > B > A$, $C + B > C + A$,

$C > A$, $C + A > B + A$, if $C + B > C + A > B + A$,

therefore $C + B > B + A$, but $B + A > C + B$,

therefore illogical,

$2 > 1$, except $C \overset{?}{=} B + A$

Under story 4, BR wrote $P > E$, $V > P$

BR then wrote

My initial reaction was to pick 1 and 2 However, 1 does not seem as logically simple as 2 and 4 do 1 seems to look at several variables when evaluating the relationship between the states (i e , fortification, economic values) Although you could end up with an ordered ranking for 1, then it is not strictly logical that $W + I > O + I$ Vanktesh does not follow a strictly additive pattern either Combinations of things are not always the sum of the parts I guess that is why I picked 2 and 4 as most similar 4 especially follows the rule that the sum of the parts equals the value of the whole 2 and 4 you could predict the rankings given basic information (except possibly for Bart's hesitancy over G vs $S + B$) (BR concluded that stories 1 and 3 were most different) The reason why 1 and 3 are dissimilar is the flip side to why 2 and 4 are similar 2 and 4 seem very logical to me They were relatively easy to decode because they followed what one expected Story 3 had some completely unexpected elements to it I do not understand how $C + B$ can be at once more preferable and less preferable to the same combinations of dishes Therefore the reason that 3 is very dissimilar is that it is not logical or predictable or understandable I read 3 at least 5 times trying to figure out where I made a mistake in interpreting the rankings But I cannot find any errors in my translation So I have to assume that it is Vanktesh that is confused If I did make a mistake I would like very much to hear back from you regarding the actual ranking of the dishes

*Level M*3 —At level *M*3, the subject understands the ambiguity of the questions requiring judgments of similarity and dissimilarity Individuals understand that there exists a multiplicity of dimensions which could provide the basis for such judgments, that is, a multiplicity of comparison frameworks The level *M*3 subject deals with this ambiguity by experimenting with a number of comparison frameworks and comparing and integrating the results of each

This reasoning can still be of a nontechnical sort, because the properties in terms of which the systems vary are not complicated ones The lattices of stories 2 and 4 are isomorphic (see fig 1), the same set of axioms applies to both Story 1 violates irreflexibility, since two states are equally preferred Story 3 violates transitivity, a more serious violation The level *M*3 subject understands that lack of transitivity means that one no longer has an order, whereas the inclusion of an equality rather than an inequality with some indeterminacy on addition still means that one has an order, although partial Therefore, the former constitutes a more serious deviation Another way to see the seriousness of the deviation is to examine what happens in the transformation from one system, *A*, to another system, *B* Such a transformation causes a loss in information to the extent that the two systems are not isomorphic This is seen when the reverse transformation is performed Thus, the result of transforming system *A* into system *B* and then back into *A* (by another transformation) must be judged from a multiplicity of frameworks Subject HC provides an example of level *M*3

Subject HC supplies the analyses that appears at the bottom of page 1067 HC then wrote

Stories 2 and 4 are the most similar because in each the same laws are established for ranking Each allows only two possible rankings (outcomes) which arise because of uncertainty as to whether one item carries a higher ranking than the sum of the two others, or vice versa If the uncertainties were settled the same way in both problems (i e , the one item sum of the other two, or vice versa) the same ranking would have been established for each A possible difference between stories 2 and 4—the fact that (not stated) in story 2, one-armed bandit machines might use two of the same color chip, whereas in story 4 it was specified that only one item of each type could be used—was eliminated by assuming that the hierarchy of preferences for each combination of two chips in story 2 represented the complete possible choice of performance Therefore, two chips of the same color were not a possible choice Refer to my above diagram [p 1067] where it is shown that the laws developed in story 4 are the same as those that govern story 2 Notations within the section for story 2 prove this in the diagrams of story 2

Story 3 is the most different because it violates both laws by which 2 and 4 are bound and because the information does not limit you to only two possible rankings Story 1 is similar to stories 2 and 4 in that the laws given are adequate for determining that there are only two possible rankings However, one of the laws used in 2 and 4 is violated and the other does not apply to 1 Story 3 is most different from stories 2 and 4 because (1) It does not set forth the same laws as those governing stories 2 and 4, in fact violating both of them, and (2) the laws governing the preference for combinations of two items are circular, i e ,

$$\begin{tikzcd} & B + Y \arrow[dl] & \\ C + B \arrow[rr, bend right] & & C + Y \arrow[ul] \end{tikzcd}$$

such that more than two possible rankings exist, even after the uncertainty of preference for chicken over baklava and yogurt, or vice-versa, are removed

*Level M*4 —Though no examples of level *M*4 have occurred in our research, so far, one can postulate this level as comprised of an idealized, maximally formalized solution to the task and example of metasystematic reasoning

At level *M*4, explicit use is made of the transformational notion Here, for instance, it may be used to show how many changes of order there are in 28 pairs of combinations that are necessary to go from one story to another and then back to the original There are other ways to assess the effects of the transformations back and forth The degree to which the information in a story may be recovered in a transformational process is a measure of its similarity The notion of an inverse transformation is used, and whether or not it can be performed without losing information is shown The properties of the system are represented in a language that is not particular to any one system Just as fully formal operational subjects in Pia-

Under story 1, HC wrote

$$\text{Illinois} > \begin{cases}\text{Utah} \\ \parallel \\ \text{Colorado}\end{cases} > 0 \qquad (1)$$

Law 1 violated, law 2 does not apply,

Utah + Colorado > Colorado + Illinois > Utah + Illinois , (2)

$$\text{Utah + Colorado + Illinois} > ? - \begin{cases} U + C > C + I > U + I > I > \begin{matrix}U\\ \parallel \\ C\end{matrix} > 0 \\ \\ I > U + C > C + I > U + I \quad \begin{matrix}U\\ \parallel \\ C\end{matrix} > 0 \end{cases} \qquad (3)$$

Under story 2, HC wrote

$B > R$ O $R > W$ O $B > W$, any, none

$1 > 2$ O $2 > 3$ O $1 > 3$ Law 2 Is this the ordering pair to pair also? (1)

Transitive property 2 Does it exist here? Yes

$B + R > B + W > R + W$	
$R > W$	$B > R$
$+B \quad +B$	$+W \quad +W$
$R + B \quad W + B$	$B + W \quad R + W$

What about the other 2 chip combinations? bracelet ring + watch Story 4 is limited in that only one of each type can be used (2)

(2 and 4 are similar)

$$R + W + B > B + R > B + W > ? \begin{cases} \rightarrow B > R + W > R \quad W > 0 \\ \rightarrow R + W > B > R > W > 0 \end{cases} \qquad (3)$$

Uncertainty happens at the last two item combination—same as story 4—ring + watch bracelet

Under story 3, HC wrote

$C > B > Y$, also $C > Y$, any > 0 (1)

$C + B > C + Y$, $C + Y > B + Y$, $B + Y > C + B$, $Y \not> C$ therefore , (2)

law 1 OK , law 1 OK , law 1 does not apply ,

$? \rightarrow C > B + Y$ or $B + Y > C$

$$C + B + Y > C + B > C + Y > ? \begin{cases} \rightarrow C > B + Y > C + B > B > Y > 0 \\ \rightarrow B + Y > C + B > C > B > Y > 0 \text{ already used} \end{cases} \qquad (3)$$

Or is it 2*C*—when has money, always 2 dishes Illogical unless does not care for baklava at all—may not—creature of habit, saves money, just *C* Transit ve property—or whatever the proper name for it is—is not working here

Under story 4, HC wrote

broken gold watches (without works), scratched gold rings, broken bracelets, any > 0 $W > B > R$, uncertainty here, too, cannot rank exactly unless you know whether $W > B + R$ or $B + R > W$ Laws

$$\left.\begin{matrix} \text{If} \quad \text{right} \quad > \quad \text{left} \\ \underline{+ \text{ equal weight} + \text{equal weight}} \end{matrix}\right\} \text{additive property} \qquad (1)$$

$$\left.\begin{matrix} \text{Right still} > \text{left} \\ 1 > 2, \quad 2 > 3, \quad \text{then} \quad 1 > 3 \end{matrix}\right\} \text{transitive property} \qquad (2)$$

$$W + B + R > ? - \begin{cases} \rightarrow W + B > W + R > W > B + R > B > R \\ \rightarrow W + B > W + R > B + R > W > B > R \end{cases}$$

get's system no longer need concrete values to be assigned to the elements to which their formal operations are applied, at the level of idealized metasystematic reasoning, the subject can operate on systems independent of their specific representations The idealized level $M4$ performance, then, would consist of a general theory of systems of order relations, within the framework of which any particular order system is evaluated Properties of the axiom systems which are used to generate these systems, such as completeness, consistency, decidability, and so on, would be considered

Performance of Subjects in the Present Sample

The levels at which subjects in the present sample were categorized are shown in table 1 The relation between performance on the formal operational problem and performance on the multisystem problem is shown in table 2 All subjects passed the concrete operational (transitivity) problem, which indicates that they functioned at least at the level of simple concrete operations The relations reflected in table 2 are in accordance with expectation With only one exception, only those subjects who showed attainment of formal operations showed any level of proficiency in systematic or metasystematic reasoning

TABLE 1

PERFORMANCE LEVELS ON THE POSTFORMAL TASK

	SAMPLE			
	Undergraduate		Graduate	
LEVEL	*N*	%	*N*	%
C	9	23	4	5
F	23	59	19	27
S	5	13	21	30
*M*1	0	0	10	14
*M*2	2	5	8	11
*M*3	0	0	9	13
Total	39		71	

TABLE 2

RELATION BETWEEN PERFORMANCE ON FORMAL AND POSTFORMAL TASKS

	POSTFORMAL TASK						
FORMAL TASK	*C*	*F*	*S*	*M*1	*M*2	*M*3	Total
Concrete	1	0	0	0	0	0	1
Transitional	5	2	1	0	0	0	8
Formal	4	30	15	2	7	6	64
Total	10	32	16	2	7	6	73

Among the 41 graduate students who received multiple administrations of the multisystem problem, most subjects showed no change or a slight advance from first to second administration Twenty of the 41 showed no change, 14 advanced one level, one advanced two levels, one advanced four levels, and five declined one level This change pattern did not differ appreciably according to time that elapsed between administrations, which suggests that change was largely attributable to effects of repeated testing

Discussion

The present results support our postulation of discrete modes of cognition composed of third-order and fourth-order operations Empirical support for the validity of the proposed constructs, labeled systematic and metasystematic reasoning, is of two sorts First, performance levels of the two samples (undergraduates and graduates) are in accordance with expectation Few undergraduates show evidence of systematic or metasystematic reasoning, its incidence is considerably greater, however, among graduate students Second, performance on the problem designed to assess systematic and metasystematic reasoning shows the appropriate relation to performance on a task designed to assess formal operational reasoning Only one of 31 subjects who did not show fully formal operational reasoning exhibited any proficiency in the use of systematic or metasystematic reasoning, but not all subjects who were proficient in formal operational reasoning exhibited proficiency in systematic or metasystematic reasoning

The purpose of the present report has been to present the instrument we have developed to assess this higher-order reasoning and to describe the performance of the initial samples of subjects to whom the instrument has been administered It is, therefore, not appropriate in this report to embark on an extended discussion of the factors or conditions that may govern the development of such higher-order reasoning To some extent, fruitful speculation in this regard awaits fuller understanding of mechanisms of cognitive development (Kuhn, in press) We should comment, however, that the performance differences between the undergraduate and graduate samples in the present study almost certainly reflect a combined contribution of self-selection and differential experience

While the "general experience" that comes with increasing chronological age appears to be a sufficient condition for attainment of the earlier stages in Piaget's system, we would not expect it to be a sufficient condition for mastery of the thought operations assessed in the present work Exposure to and experience with problems that require abstract representational modes of analysis are undoubtedly necessary factors, but just how native ability, education, and experience interact in this regard is a difficult issue to address (see Commons, Richards, & Armon [in press] for additional discussion)

The caveat introduced earlier bears reiteration It is not necessarily the case, and indeed most unlikely, that all adult thought is of the form investigated here One of the pressing issues in the study of adult cognitive development, in fact, is to discover the role that formal, logical/deductive reasoning plays in the real-world thought that occurs in adulthood (Gilligan & Murphy 1979, Kuhn et al, in press, Labouvie-Vief 1982) Nevertheless, within the realm of logical hypothetical/deductive thought studied by Piaget, levels of reasoning beyond Piaget's formal operations in our view must be of the general form of third-order and fourth-order operations that we have outlined It is too restrictive to say that systematic and metasystematic operations are limited to the domains of mathematics and science Numerous other disciplines, such as literature, history, or anthropology, entail the evaluation of systems within a multisystem framework It may be the case, however, that systematic and metasystematic reasoning is limited to the domain of formal, abstract, as opposed to everyday, thought, in contrast to Piaget's formal operations which are discernible in everyday thinking Clearly, a good deal of further work will be required to establish the variety of forms that the systematic and metasystematic reasoning identified in the task presented here may take

References

Arlin, P Cognitive development in adulthood a fifth stage? *Developmental Psychology*, 1975, **11**, 602–606

Bickhard, M On necessary and specific capabilities in evolution and development *Human Development*, 1979, **22**, 217–224

Commons, M L, Richards, F A, & Armon (Eds) *Beyond formal operations late adolescent and adult cognitive development* New York Praeger, in press

Fakouri, M "Cognitive Development in Adulthood a fifth stage?" a critique *Developmental Psychology*, 1976, **12**, 472

Gilligan, C, & Murphy, M Development from adolescence to adulthood the philosopher and the dilemma of the fact In D Kuhn (Ed), *New directions for child development* Vol **5** *Intellectual development beyond childhood* San Francisco Jossey-Bass, 1979

Inhelder, B, & Piaget, J *The growth of logical thinking from childhood to adolescence* New York Basic, 1958

Kuhn, D On the dual executive and its significance in the development of developmental psychology In D Kuhn & J Meacham (Eds), *On the development of developmental psychology* Basel Karger, in press

Kuhn, D, Pennington, N, & Leadbeater, B Adult thinking in developmental perspective the sample case of juror reasoning In P Baltes & O Brim (Eds), *Life-span development and behavior* Vol **5** New York Academic Press, in press

Labouvie-Vief, G Beyond formal operations uses and limits of pure logic in life-span development *Human Development*, 1980, **23**, 141–161

Labouvie-Vief, G Growth and aging in life-span perspective *Human Development*, 1982, **25**, 65–78

Riegel, K Dialectic operations the final period of cognitive development *Human Development*, 1973, **16**, 346–370