The purpose of the present paper is to explore the possibility that modes of systematic reasoning can be identified that are qualitatively distinct from, and logically more complex than, the form of reasoning characterized by Inhelder and Piaget (1958) as "formal operational." In this endeavor, we have made use of Piaget’s system of successive stages of logical operations only in its most general form. In other words, Piaget’s stages are regarded as successive levels of cognitive representation, in this sense, the stages are not subject to any form of empirical verification, but rather constitute a conceptual description of forms of cognition (Bickhard 1979). Thus, sensory-motor operations are defined as actions on material objects, symbolic operations as actions on symbolic representations of material objects (e.g., the concrete operational organization of symbolically represented objects into classes or relations), and formal operations as operations on the above-mentioned operations (of classification or relation), for example, the individual organizes classes (of elements) according to relations that obtain among them. The object of the present work was to inquire whether levels of higher-order operations could be identified and, if so, to establish the particular form they would take in the cognitive performance of the individual.

There are several existing attempts in the developmental literature to postulate "postformal operational" stages of cognition, notably those of Riegel (1973) and Arlin (1975). The major problem with these formulations is that it is uncertain the extent to which these various postulated modes of cognition constitute a qualitatively distinct structure or stage of reasoning, related in a hierarchical manner to formal operations, or, alternatively, whether they are modes of cognition that develop parallel to Piaget’s stage sequence. Forms of Arlin’s stage of "problem finding," for example, are likely to be present during the stages of concrete and formal operations (Fakoun 1976), thus, the stage of formal operations cannot be regarded as the necessary but insufficient condition for the stage of problem finding, as it would need to be if the two were to be related in a hierarchical manner. A similar consideration applies to Riegel’s "dialectic operations." As Riegel (1973) noted, dialectical operations are potentially present in some form at all of Piaget’s stages.

It is our view, then, that if forms of advanced cognition are to be identified that make...
reference to, and build on, Piaget's sequence of stages of cognition, they must take the general form of the higher-order operations on operations on operations (and possibly beyond) referred to above. To make this claim is not to assert that all forms of adult cognition necessarily fall within such categories. Indeed, current research and theory in adult cognitive development run strongly counter to such a claim (Kuhn, Pennington, & Leadbeater, in press, Labovwe-Vief 1980, 1982). Rather, our claim is that if one wishes to postulate more advanced forms of logical/mathematical reasoning of the type studied by Piaget, that build on the forms of reasoning that constitute his existing stage sequence, then this more advanced reasoning must take the general form we have indicated.

What might such higher-order operations look like in the cognition of an individual? We can refer to the second-order, or formal, operations in Piaget's system as reflecting "inter-relational thinking." That is, reasoning is based on ordered relations of classes or relations, the individual executes operations on these classes or relations. For example, the individual may formulate and test hypotheses about the relations that obtain among two classes of objects. In executing these formal operations, however, the individual does not self-consciously represent, reflect, or operate on the system as a whole. We postulate, then, a possible set of third-order operations, termed "systematic operations," which consist of exhaustive operations on classes or relations of classes or relations, forming systems. We further postulate the possibility of a set of fourth-order operations, termed "metasystematic operations," which consist of operations on systems.

Systematic operations apply to the entire set of constituents of a system (that itself is made up of operations on operations). For example, they might consist of the coordination of iterative operations such as $\Sigma_4$ or the coordination of abstract representations of operations, such as $a \circ b$, $a \bullet b$, to form systems. Such systematic descriptions formally represent the properties of operations at the formal operational level.

Metasystematic operations are cognitions about systems. They are required in the formation of a framework (or "metasystem") for comparing and contrasting systems with one another. The relationship of one system to another such system is expressed as a metatheory and is found by comparing axioms, theorems, or other limiting conditions of systems within the framework of a "super-system" that contains all of the variant systems. Metasystematic reasoning is defined as the set of operations necessary to construct the supersystem and to execute the analysis of the systems contained therein.

An example of metasystematic reasoning is found in that aspect of Einstein's general theory of relativity that deals with the coordination of inertial and gravitational mass. Prior to Einstein's formulation, there was one system for describing inertial mass and a separate one for describing gravitational mass. Each separate system included systematic representations of properties of formally defined relations between variables. Inertial mass was the property of the system that described a body's resistance to acceleration. Gravitational mass was a property of another system that described the weight of a body in a given gravitational field. In fact, since the same constant for mass represents both the inertia and the weight of a body, it follows that it is impossible to discover by experiment which of the following is true. The motion of a given system of coordinates is straight and uniform, and the observed effects within the system are due to a gravitational field, or the system of coordinates is uniformly accelerated and the observed effects within the system are due to inertia. Recognition of an equivalence between the two cases constitutes the equivalence principle of general relativity theory. This principle states that the inertial system is isomorphic (contains the same structure and elements) to the gravitational system, that is, any relationship that is true in one is true in the other. Whether the same property appears as weight or as inertia depends on which description of the coordinate systems is employed, that is, motion in a gravitational field appears in relation to an inertial system of coordinates, while motion in the absence of a gravitational field appears in a coordinate system that is accelerated.

This principle requires a coordination of two distinct systems, each generated by systematic operations of the inertial system and the gravitational system. The cognitive operations that effect this coordination are at a "metasystematic" level, distinct from that of the operations applied within either of the individual systems.

The above example serves to illustrate the irreducibility of the higher-order operations to operations of the next lower order. Operations at the metasystematic level must be expressed in the language of metalogic, or its psychological equivalent, because statements about the relation between systems cannot be reduced to statements about the properties of the relations.
within any single system. Similarly, statements about systems properties cannot be contained within any of the individual systems. Hence, systematic operations cannot be reduced to formal operations because the formal operational systems themselves do not contain descriptions of the system. These formal operations are not rich enough to describe their own properties. If this were possible, Russell’s paradox (the sentence, “This sentence is false”) would not exist. In other words, a property of a system cannot be described by a proposition within a system.

**Method**

**Task**

In the problem we developed to assess systematic and metasystematic reasoning, the subject was asked to compare and contrast four systems, each comprised of a set of asymmetric relations. Two similar forms of the problems were constructed. One form of the problem and the instructions given to the subject are shown below:

Here are four stones. After you read them, you will answer questions on which are most similar and which are most different. Use the “greater than” symbol “>” to indicate the order of things. For example, indicate “Brown prefers Oregon over Texas, by O > T Only attend to order

1) On counter earth, Richard Reagan has been elected President of the United States. As a gesture of gratitude to the people of his home state, California, Reagan has convinced them to leave California and either wander around with no state, or take a combination of one, two, or three of the following states Oregon, Washington, and Indiana. Reagan thinks that the economic value of Oregon and Washington are equal, and that the value of either is less than that of Indiana. The boundary and passport bill which he submitted to Congress, requiring the barbed wiring and mining of the boundaries at considerable expense, did not pass. Instead, Congress specified that the funds asked for in that bill were to go to the states selected to be used as states saw fit. Only Washington and Oregon have common boundaries and, therefore, a union of these two states would receive a smaller amount of the fortification funds. Therefore, even with the value of their combined economies, Reagan thinks that the pair Oregon and Washington is less valuable than the pairs Oregon and Indiana or Washington and Indiana. Since Indiana makes boats which can be used on the Columbia River, Reagan thinks that the economic benefits of pairing Washington and Indiana are slightly greater than the benefits of pairing Oregon and Indiana. Reagan thinks that three states are worth more than any combination of 0, 1, or 2 states, and any 2 states are worth more than 0 or 1 states, except he cannot decide whether it is worth more, Indiana or the pair Oregon and Washington.

2) Bad Bart ambles into the local casino and converts his gold watch into chips of the following colors: silver, bronze, and gold. Bart likes to play the one-chip candy machines. He likes the chips in the following order: gold better than silver, silver better than bronze, and gold better than bronze, and any chip over none. Bart also likes to play the one-armed bandit machines which use combinations of two chips. He likes the two-chip combinations in the following order: golds and silvers first, golds and bronzes second, and silvers and bronzes third. With one exception, Bart knows he likes to play with any two chips over one or none, he is not sure about a gold versus a silver and a bronze. Because Bad Bart likes to play with three chips best, there is one machine that he likes better than any other the washing machine, and it takes a silver, a bronze, and a gold chip.

3) In Madras, India, V P Vanktesh, a man of habit and variable income, has a favorite restaurant. Although his tastes never vary, the food he can afford does. Of the three foods the restaurant serves, he prefers curry, birami, and alu parathra, in that order. Also, he likes curry better than alu paratha, and anything better than nothing. When V P has more money he buys two dishes, except it is not known whether he would choose the curry over both the birami and the alu paratha. He likes the combination of curry and birami better than curry and alu paratha, and he also likes curry and alu paratha better than birami and alu paratha, and birami and alu paratha better than curry and birami. Although a temperate man, at festivals, given the means, he has all three dishes instead of any single dish or pair.

4) A jeweler has three boxes, the first containing different kinds of broken 18-carat gold necklaces, the second various scratched earrings, and the third different kinds of 18-carat gold pins that are broken. He keeps the old jewelry because he occasionally uses the gold. To get the approximate amount of gold he needs, he weighs and then melts down a combination of objects. To do the weighing he uses a simple balance-beam scale. It consists of a beam that pivots in the middle and two pans hanging from each end of the beam, equidistant from the pivot. The beam is level when the pans are empty. The jeweler can place combinations of from 0 to 3 object types into one pan, but never more than one of each object type in a pan. Begin...
nning with empty pans, he notices that whenever
he puts any combination containing at least one
object into a pan, that pan sinks down, indicating
that it is heavier than the empty pan. Using this
same method, he finds that any pin is heavier than
any earring. Necklaces are always heavier than
pins. He has discovered two rules that reduce how
many combinations he needs to try to find out how
the weights of the combinations are ordered. First,
he notices that if the combination in the right pan
is heavier than the combination in the left pan,
and a single object type not already in either of
the pans is added to both pans, the right pan re-
 mains heavier than the left. Secondly, if he weighs
three combinations of objects, he finds that the fol-
lowing is always true. If the first combination is
heavier than the second, and the second heavier
than the third, then the first is heavier than the third.

Space was left after each story for subjects
to make notations. On a separate page, the follow-
ing instructions were presented.

Now that you have read the stones and are
familiar with them, answer the questions below.
Make your comparisons on the basis of properties
of the orderings found in each story, in the best
and most complete way that you can. Write out all
the comparisons that you can in a systematic way.
Use symbols to represent the order of things, and
explain what the symbols stand for. Also include
an explanation in English. You may want to use a
Oregon chain, for example, \( \downarrow \), in addition to an order-
ing, Oregon > Texas. Then explain what are the
most important similarities and differences in the
stones, and explain how you arrived at deciding
the relative importance of these similarities and dif-
ferences. You may refer to more than one frame-
work. Make sure to give the strongest, most thor-
ough, broad, inclusive, and complete explanations
possible for the similarities and differences. It is
necessary that you show all of your work in forming
the orders and making the comparisons, as well as
your commentary in English.

The following form was used for subjects
to provide their answers:

1. Which of the stones are the most similar?
1 and 2, 2 and 3, 1, 2, and 3, 1, 3, and 4
1 and 3, 2 and 4, 1, 2, and 4, 1, 2, 3, and 4
1 and 4, 3 and 4, 2 and 3, and 4

2. Which of the stones differ the most from the
ones you listed as similar in (1), even though
they may have many characteristics in com-
mon?

---

Space was provided for subject's answer.

Each of the four systems within the prob-
lem consists of a finite, partially ordered, com-
mutative semigroup, with a binary operation for
combining objects and a partial order relation
defined among all possible combinations of ele-
ments. The systems were presented as stones
about which combinations of three objects, \( a, b,
\) \( c \), were greater than others. The combinations
could include no item, single items, or two or
three items. In the first three stones, for the
most part, the orderings of combinations of ob-
jects were explicitly stated. One object is pre-
ferred over another, or over none at all, for
example, \( a \) over none, \( c \) over \( b \), \( b \) over \( a \),
and so forth. Some pairs are preferred over others,
for example, \( b + c \) over \( a + b \), or the complete
unit is preferred over a pair, for example,
\( a + b + c \) over \( b + c \). In the fourth story, the
structure of the order was stated in proposi-
tional form which did not provide direct access
to the orderings of the elements, these had to
be derived by the subject.

An alternate form \( B \) was presented to a
portion of the sample. Forms \( A \) and \( B \) were
identical except for the structure reflected in
story 1 (see fig 1) and the names of the ele-
ments in each of the stones, for example, the
names of the states in story 1 and the foods in
story 3.

The subject was allowed an unlimited
amount of time. The average amount of time
spent was 1 hour, with an approximate range
from 30 min to 2 hours.

Two additional problems were presented
to a portion of the subjects (73 of the 110) a
simple transitivity problem (requiring concrete
operations) and a version of Inhelder and Pia-
get's (1958) pendulum problem (requiring for-
mal operations). Problem order was counter-
balanced. It was hypothesized that performance
on the three problems would show a hierarchi-
cal pattern, that is, no subject would master the
pendulum problem who did not master the tran-
sitivity problem, and no subject would master

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1 This problem is used with the permission of the Dare Association, Inc.
The 110 subjects were 39 undergraduates and 71 graduate students attending one of several private universities in the Northeast. The mean ages for the groups are 20.6 and 26.1 years, respectively. All participated on a volunteer basis. The multisystem problem was administered a second time to 41 of the 71 graduate students, directly following the initial administration for 22 of them, and 8 months following the initial administration for 19 of them. (Half of the students received form A first, the other half form B first.)

Results

Analysis of subjects’ protocols suggested six distinct levels of response. The scoring system was developed based on an intensive analysis of 25 of the protocols, guided by the theoretical perspective set forth in the introduction. It was then applied to the remaining 85. Fifty-five of the 85 protocols were evaluated independently by raters 1 (first author) and 2 (research assistant), 61 of the 85 protocols by raters 1 and 3 (research assistant). There was 76% and 62% agreement, respectively. Differences were resolved by discussion and a final level assigned to each protocol.

Level C — The initial level, labeled C, resembles the mode of thinking termed concrete operational in Piaget’s system. Subjects categorized in this level base their judgments of similarity/dissimilarity on superficial features of the stones rather than on order relations (either within or across stones), as the task instructions direct. If order relations are attended to at all, this attention is limited to a representation of the discrete order relations explicitly given in the story. The subject does not perform any operations on these elements, to derive additional order relations or more general properties of the system (story). Subject RA provides an example (The examples to follow are based on both form A and form B, so that letters may not always match the stones given earlier.)
Under story 1, RA wrote
\[ U = C, \ U < I, \ C < I, \ CI > UI, \]
\[ (I \geq UC), \ 3 > 2 > 1 \]

Under story 2, RA wrote
He likes \( B > R, \ R > W, \ B > W, \)
\[ (3 > 2 > 1), \ BR > BW > RW, \]
\( (B > RW)? \)

Under story 3, RA wrote
He prefers \( CK > Bak > Yog, \)
\( Ck - Ba > Ch - Yo, \)
\( Ch - Yo > Ba - Yo, \)
\( Ba - Yo > Ck - Ba?, \)
\[ (3 > 2 > 1) (Ch > Ba - Yo)? \]

Under story 4, RA wrote
\( Br > Rl, \ W a > Br, \ (W a > Rl), \)
\[ (3 > 2 > 1) \text{comb, right pan, left pan} \]

RA then wrote “Stories 1, 2, and 3 are most similar because in the first three stories a person’s opinion is involved. He thinks, or he likes, or he prefers, a particular object or a combination of objects, and if another person were to make a choice on what he would prefer of such combinations, the combinations would be likely to change (e.g., some people may like white chips better than blue). At the same time, story 4 is based on facts, and if the jeweler were replaced, the facts about the combinations would still be true.” Note that RA does not link information from discrete order relations into a higher-order unit or chain (though we know RA is capable of a transitivity inference) \( B > R, R > W, \) and \( B > W \) are not integrated into a unified order relation \( B > R > W \)

**Level F**—This level is postulated to be equivalent to the level of formal operations in Piaget’s sequence (Piaget’s levels II A & B, see Inhelder & Piaget [1958]). Order relations within stories are operated on in a systematic manner, but the entire system is not regarded (operated on) as a single entity having characteristic properties that may be compared with the properties of other systems. Thus, the subject’s attempt to relate the stories with respect to their similarity/dissimilarity is limited to establishing that single elements or two-element order relations map to some degree from one of the first three stories to another Subject LR provides an example. Under story 1, LR wrote
\[ I > W, \ O, W > W + I > OI, \]
\[ 3 > 2 O + W + I, \text{ uncertainty } I ? O, W \]

Under story 2, LR wrote
\[ g > s > b > n \text{one exception}, \]
\[ gs > gb > sb, \text{ one uncertainty } g ? b, s \]

Under story 3, LR wrote
\[ c > b > a > n \text{one case}, \]
\[ \text{ uncertainty in one case } b, a ? c \]

Under story 4, LR wrote
\[ n > p > e > n \text{one case}, \]
\[ \text{ no uncertainty, no pairing} \]

LR then wrote “1 and 3 are similar because both individuals prefer any 3 to any 2, any 2 to any 1, and any 1 to any none, with one uncertainty. In story 1, Reagan does not know if \( I > O, W, \) and in story 3, Vanktesh does not know if \( B, A > C \) One difference is that the two singles are equal to each other in story 1 \( (W = O) \), whereas this is not true of story 3 \( (C > B > A) \) The other difference is that the Californians may or may not benefit most from taking \( I + O \) over \( W, \) whereas Vanktesh definitely would take all three.”

**Level S**—Responses at this level reflect the application of what we have termed systematic reasoning. At this level, subjects clearly show that they understand that the logical structure of each story must be examined as an integral whole or structure. In representing the structure of each story, the subject may choose one of two possible courses. A schematic representation of each of the systems can be generated and these representations compared with respect to their deviations from one another. Alternatively, the subject can represent the systems on the basis of the axioms that do or do not characterize each of the systems. The two methods yield equivalent results. Use of these representations is taken as evidence that the subject perceives the story as a system, that is, a coherent whole which determines the internal pattern of relations across its elements. When systematic operations are fully consolidated, full representations are constructed. There is no evidence for the presence of a framework for intersystem comparison, cognition is focused on intrasystem analysis. Subject Commons, Richards, and Kuhn 1063
NJ provides an example. Under story 1, NJ wrote:

\[ O = W, O < I, W < I, O + W > O + I, \]
\[ O + W > W + I, W + I > O + I, \]
\[ O + W + I > W + I, \]
\[ O + W + I > O + I, \]
\[ O + W + I > W + O, O + I > W, \]
\[ O + W > I, I + W > O \]

Under story 2, NJ wrote:

\[ G > S, S > B, G > B, G > O, \]
\[ B > O, S > O, G + S > G + B > \]
\[ S + B, G > B + S, G + B > S, \]
\[ G + S > B, G + S + B > G + S > \]
\[ G + B > S + B \]

Under story 3, NJ wrote:

\[ C > B > A, C > A, C > O, A > O, \]
\[ B > O, C > B + A, C + B > A, \]
\[ C + A > B + A, B + A > C + B, \]
\[ A + B + C > B + A, A + C, \]
\[ B + C, A, B, C \]

Under story 4, NJ wrote:

\[ N > O, E > O, P > O, P > E, \]
\[ N > P \]

(Note that story 4 is incompletely represented.)

NJ then wrote: “1 and 2 both use the technique of comparing items in regard to their relative merit on a one-to-one basis, then in pairs. In addition, the relative worth of combination versus single item and versus zero is analyzed. 1 and 2 also follow the transitive law of geometry, that is \( x > y \) and \( y > z \), then \( x > z \). The law holds for 1 and 2, whereas it fails in example 3. In 3, \( B + A > C + B \), then by taking away \( B, A > C \), but this contradicts what we are told before. I realize, however, that Mr. Vanktesh’s preferences need not follow mathematical laws and logic, he may indeed prefer the \( B + A \) combination better. I point this out because I felt this makes 3 less similar to 1 than 2 was to 1.” NJ’s reasoning is characterized by an effort to describe differences across stories. Stories are compared in a pairwise fashion in this effort, evidence for the lack of a systematic framework for comparison.

**Level M1** — At this level the first evidence of metasystematic operations appears. Comparisons across stories are based on variations of the structural properties, implicitly indicating the possibility of conceptualizing one story as the transformation of another story. At level M1, the subject generates a much more complete set of lattices and/or axioms to represent or characterize the stories. However, some axioms or orderings necessary to complete the analysis are clearly missing. Subject CC serves as an example by providing only a partial analysis of story 4 (leaving out the details of its ordering and leaving out the single combinations of the elements). Subject CC’s analysis is as follows:

Under story 1 CC wrote:

\[ I > O = W, O + W + I > O + W > W + I > O + I > I = O = W \]

where do these fit?

\[ O + W > O + I, W + I, \]
\[ W + I > O + I, I > O + W \]

Under story 2 CC wrote:

\[ G > S > B > \text{none}, GSB > GS > GB > SB 
\]
\[ = G > S > B > \text{none}, GS > GB > SB \]

Under story 3 CC wrote:

\[ C > B > A > \text{nothing}, C + B > C + A, \]
\[ C > A > \text{none}, C + A > B + A, \]
\[ C > B + A, B + A > C + B, \]
\[ C + B + A > C + B > C + A > B + A \]

Under story 4 CC wrote:

\[ N > P > E > L, NP > NE, NP > NE > PE, PR PE, VP > PE, PN PE, NE > PE \]
CC then wrote "[In stories 1 and 3], there is a basic similarity in the structure combinations of three, combinations of two, one, none. However, in both stories, there is a degree of uncertainty as to precise order of the combinations. In story 1, $O + W > W > I > O + I > I = O = W$, and also the ranking of $I$ versus $O + W$ is uncertain. Therefore, if $I$ is greater than $O + W$, then the preferences become 'circular' as in story 3. If $I = O + W$, then again the preferences are circular $O + W = I > W + I > O + I > I$. In story 3, the circular nature of the combinations is spelled out $C + B + A > C + B > C + A = B + A > C + B > B > A$.

Story 4 differs the most. Both 2 and 4 have very clear relationships. Though the relationships in story 4 are not explicitly spelled out, there is enough information to deduce that a combination of 3 > combinations of 2 > singles. In story 2, the relationship is also the same, though there is slight uncertainty as to the relationship between $SB$ and $G$. Nonetheless, this uncertainty is not enough to change the order of the relationships the way it does in stories 1 and 3."

It is clear that CC's reasoning and representations could be carried further. On the other hand, CC does use "circularity" and uncertainty of order to refer to the integrated structures as wholes, and she generates lattices for three stones. Structure is compared across stones, even though incorrectly. The use of circularity and "uncertainty" indicates the existence of a common "source structure" whose properties are altered to produce the different instantiations of structure found in the stories.

**Level M2**—This is the second level in the emergence of truly metasystematic operations. This level, and the two which follow it, are progressive steps in the consolidation and organization into a whole, of operations involved in comparisons across systems. Subjects at this level have, either explicitly or implicitly, full and integrated representations of the systems of ordered relations reflected in each of the four stories (Systematic thinking is now fully consolidated). These representations are used to check (again either explicitly or implicitly) their assertions about the systems in a systematic and complete fashion. These subjects choose a single property which is appropriate for comparing the integrated structures of the stories, as opposed to one that would enable comparisons of parts only. For example, the subject might compare the stories on the basis of either additivity or transitivity properties. Such properties are used to construct comparison frameworks. For a subject who represents the systems underlying the stories graphically, the comparison framework may consist simply of a set of dimensions along which the physical drawings of the systems are tested for resemblance. A subject may imply that he or she has generated complete orderings in all four stories without actually showing the work. Thus, the simple assertion that "stories 2 and 4 follow the law of additivity while 1 and 3 do not," is enough to classify a subject at level M2 (In protocols classified at levels M1 or S, in contrast, there exists clear evidence that certain combinations, i.e., relations between elements, were not considered in generating the representation). Subject BR provides an example of level M2.

Subject BR represented the stories as follows:

**Under story 1, BR wrote**

- $O = W$ common boundaries economically
- $O < I, I > O, O + W > fortifying, H + I > O + I,
- $W < I, I > W, O + I < fortifying, S > pair,
- $W + I < fortifying, pair > single$

**Under story 2, BR wrote**

- $G > S, G + S > B > S + B, 3 chips > 2 chips$,
- $S > B, 1 chip > better than 1 chip$,
- $G > B, except G = 5 + B$.

**Under story 3, BR wrote**

- Confused Indian or exam taker?
- $C > B > A, C + B > C + 4$,
- $C > A, C + A > B + A$, if $C + B > C + 4 > B + 4$,
- therefore $C + B > B + 4$, but $B + 4 > C + B$,
- therefore illogical,
- $2 > 1$, except $C = B + A$.

**Under story 4, BR wrote**

- $P > E, \forall > P$. «
BR then wrote

My initial reaction was to pick 1 and 2. However, 1 does not seem as logically simple as 2 and 4 do. It seems to look at several variables when evaluating the relationship between the states (i.e., fortification, economic values). Although you could end up with an ordered ranking for 1, then it is not strictly logical that \( W + I > O + I \). Vanktesh does not follow a strictly additive pattern; either combinations of things are not always the sum of the parts. I guess that is why I picked 2 and 4 as most similar. 4 especially follows the rule that the sum of the parts equals the value of the whole. 2 and 4 you could predict the rankings given basic information (except possibly for Bart’s hesitancy over C vs S + B) (BR concluded that stories 1 and 3 were most different.) The reason why 1 and 3 are dissimilar is the flip side to why 2 and 4 are similar. 2 and 4 seem very logical to me. They were relatively easy to decode because they followed what one expected. Story 3 had some completely unexpected elements to it. I do not understand how \( C + B \) can be at once more preferable and less preferable to the same combinations of dishes. Therefore, the reason that 3 is very dissimilar is that it is not logical or predictable or understandable. I read 3 at least 5 times trying to figure out where I made a mistake in interpreting the rankings. But I cannot find any errors in my translation. So I have to assume that it is Vanktesh that is confused. If I did make a mistake I would like very much to hear back from you regarding the actual ranking of the dishes.

**Level M3** — At level M3, the subject understands the ambiguity of the questions requiring judgments of similarity and dissimilarity. Individuals understand that there exists a multiplicity of dimensions which could provide the basis for such judgments, that is, a multiplicity of comparison frameworks. The level M3 subject deals with this ambiguity by experimenting with a number of comparison frameworks and comparing and integrating the results of each.

This reasoning can still be of a nontechnical sort, because the properties in terms of which the systems vary are not complicated ones. The lattices of stories 2 and 4 are isomorphic (see fig 1), the same set of axioms applies to both. Story 1 violates transitivity, since two states are equally preferred. Story 3 violates transitivity, a more serious violation. The level M3 subject understands that lack of transitivity means that one no longer has an order, whereas the inclusion of an equality rather than an inequality with some indeterminacy on addition still means that one has an order, although partial. Therefore, the former constitutes a more serious deviation. Another way to see the seriousness of the deviation is to examine what happens in the transformation from one system, A, to another system, B. Such a transformation causes a loss in information to the extent that the two systems are not isomorphic. This is seen when the reverse transformation is performed. Thus, the result of transforming system A into system B and then back into A (by another transformation) must be judged from a multiplicity of frameworks. Subject HC supplies an example of level M3.

Subject HC supplies the analyses that appears at the bottom of page 1067. HC then wrote

Stories 2 and 4 are the most similar because in each of the same laws are established for ranking. Each allows only two possible rankings (outcomes), which arise because of uncertainty as to whether one item carries a higher ranking than the sum of the two others, or vice versa. If the uncertainties were settled the same way in both problems (i.e., the one item sum of the other two, or vice versa) the same ranking would have been established for each. A possible difference between stories 2 and 4—the fact that (not stated) in story 2, one-armed bandit machines might use two of the same color chip, whereas in story 4 it was specified that only one item of each type could be used—was eliminated by assuming that the hierarchy of preferences for each combination of two chips in story 2 represented the complete possible choice of performance. Therefore, two chips of the same color were not a possible choice. Refer to my above diagram [p. 1067] where it is shown that the laws developed in story 4 are the same as those that govern story 2. Notations within the section for story 2 prove this in the diagrams of story 2.

Story 3 is the most different because it violates both laws by which 2 and 4 are bound and because the information does not limit you to only two possible rankings. Story 1 is similar to stories 2 and 4 in that the laws given are adequate for determining that there are only two possible rankings. However, one of the laws used in 2 and 4 is violated and the other does not apply to 1. Story 3 is most different from stories 2 and 4 because (1) it does not set forth the same laws as those governing stories 2 and 4, in fact violating both of them, and (2) the laws governing the preference for combinations of two items are circular, i.e.,

\[
\begin{align*}
& \text{such that more than two possible rankings exist, even after the uncertainty of preference for chicken over baklava and yogurt, or vice-versa, are removed.} \\
& \quad \text{Level M4 — Though no examples of level M4 have occurred in our research, so far, one can postulate this level as comprised of an idealized, maximally formalized solution to the task and example of metasystematic reasoning.}
\end{align*}
\]
At level \( M4 \), explicit use is made of the transformational notion. Here, for instance, it may be useful to show how many changes of order there are in 28 pairs of combinations that are necessary to go from one story to another and then back to the original. There are other ways to assess the effects of the transformations back and forth. The degree to which the information in a story may be recovered in a transformational process is a measure of its similarity. The notion of an inverse transformation is used, and whether or not it can be performed without losing information is shown. The properties of the system are represented in a language that is not particular to any one system just as fully formal operational subjects in Pia-

Under story 1, HC wrote

\[
\text{Illinois} > \text{Utah} > 0
\]

\[
\text{Colorado} > \text{Utah} > 0
\]

Law 1 violated, law 2 does not apply.

\[
\text{Utah} + \text{Colorado} > (\text{Colorado} + \text{Illinois}) > \text{Utah} + \text{Illinois}
\]

Under story 2, HC wrote

\[
B > R > B > H > B > W > R + H
\]

Law 2: Is this the ordering pair to pair also?

Transitive property 2: Does it exist here?

What about the other 2 chip combinations?

\[
R > W + B > H + W + B + H + W > R + H + B + W + B + W + R + H
\]

\[
R + W + B > B + R > B + W > R + H + B + W
\]

\[
1 > 2 > 3 > 1 > 3
\]

2 and 4 are similar.

Under story 2, HC wrote

\[
\text{C} > \text{B} > \text{Y} ,\ \text{also} \ C > Y ,\ \text{any} > 0
\]

\[
\text{C} + \text{B} > \text{C} + \text{B} + 1 ,\ \text{B} + \text{Y} > \text{C} + \text{B} , \ 1 > \text{C}
\]

law 1 OK.

\[
\text{C} + \text{B} > \text{C} + \text{B} + 1 ,\ \text{B} + \text{Y} > \text{C} + \text{B} , \ 1 > \text{C}
\]

law 1 does not apply.

\[
? \rightarrow \text{C} > \text{B} + \text{Y} \ \text{or} \ B + \text{Y} > \text{C} + \text{B} + \text{B} + \text{B} > \text{B} + \text{Y} > \text{C} + \text{B} + \text{B} + \text{B} > \text{B} + \text{Y}
\]

\[
\text{C} + \text{B} + 1 > \text{C} + \text{B} + \text{C} + \text{Y} > \text{B} + \text{Y} + \text{C} + \text{B} + \text{B} + \text{Y} + \text{C} + \text{B} + \text{B} > \text{B} + \text{Y}
\]

Or is it 2C—when has money, always 2 dishes. Illogical unless does not care for baklava at all—may not—creation of habit, saves money, just C. Transitive property—or whatever the proper name for it is—is not working here.

Under story 4, HC wrote

broken gold watches (without works), scratched gold rings, broken bracelets, any > 0 \( H > B > R \), uncertainty here, too, cannot rank exactly unless you know whether \( H > B + R \) or \( B + R > W \). Laws

\[
\text{If right} > \text{left} \rightarrow \text{additive property}
\]

\[
\text{Right still} > \text{left} \rightarrow \text{transitive property}
\]

\[
\]

| Commons, Richards, and Kuhn | 1067 |
get's system no longer need concrete values to be assigned to the elements to which their formal operations are applied, at the level of idealized metasystematic reasoning, the subject can operate on systems independent of their specific representations. The idealized level M4 performance, then, would consist of a general theory of systems of order relations, within the framework of which any particular order system is evaluated. Properties of the axiom systems which are used to generate these systems, such as completeness, consistency, decidability, and so on, would be considered.

**Performance of Subjects in the Present Sample**

The levels at which subjects in the present sample were categorized are shown in Table 1. The relation between performance on the formal operational problem and performance on the multisystem problem is shown in Table 2. All subjects passed the concrete operational (transitivity) problem, which indicates that they functioned at least at the level of simple concrete operations. The relations reflected in Table 2 are in accordance with expectation. With only one exception, only those subjects who showed attainment of formal operations showed any level of proficiency in systematic or metasystematic reasoning.

**Table 1**

<table>
<thead>
<tr>
<th>LEVEL</th>
<th>UNDERGRADUATE</th>
<th>GRADUATE</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>9</td>
<td>4</td>
</tr>
<tr>
<td>F</td>
<td>23</td>
<td>19</td>
</tr>
<tr>
<td>S</td>
<td>5</td>
<td>21</td>
</tr>
<tr>
<td>M₁</td>
<td>0</td>
<td>10</td>
</tr>
<tr>
<td>M₂</td>
<td>2</td>
<td>8</td>
</tr>
<tr>
<td>M₃</td>
<td>0</td>
<td>9</td>
</tr>
<tr>
<td>TOTAL</td>
<td>39</td>
<td>71</td>
</tr>
</tbody>
</table>

**Table 2**

<table>
<thead>
<tr>
<th>POSTFORMAL TASK</th>
<th>C</th>
<th>F</th>
<th>S</th>
<th>M₁</th>
<th>M₂</th>
<th>M₃</th>
<th>TOTAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>CONCRETE</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>TRANSITIONAL</td>
<td>5</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>8</td>
<td>14</td>
</tr>
<tr>
<td>FORMAL</td>
<td>4</td>
<td>30</td>
<td>15</td>
<td>2</td>
<td>7</td>
<td>64</td>
<td>73</td>
</tr>
<tr>
<td>TOTAL</td>
<td>10</td>
<td>32</td>
<td>16</td>
<td>2</td>
<td>7</td>
<td>64</td>
<td>73</td>
</tr>
</tbody>
</table>

Among the 41 graduate students who received multiple administrations of the multisystem problem, most subjects showed no change or a slight advance from first to second administration. Twenty of the 41 showed no change, 14 advanced one level, one advanced two levels, one advanced four levels, and five declined one level. This change pattern did not differ appreciably according to time that elapsed between administrations, which suggests that change was largely attributable to effects of repeated testing.

**Discussion**

The present results support our postulation of discrete modes of cognition composed of third-order and fourth-order operations. Empirical support for the validity of the proposed constructs, labeled systematic and metasystematic reasoning, is of two sorts. First, performance levels of the two samples (undergraduates and graduates) are in accordance with expectation. Few undergraduates show evidence of systematic or metasystematic reasoning, its incidence is considerably greater, however, among graduate students. Second, performance on the problem designed to assess systematic and metasystematic reasoning shows the appropriate relation to performance on a task designed to assess formal operational reasoning. Only one of 31 subjects who did not show fully formal operational reasoning exhibited any proficiency in the use of systematic or metasystematic reasoning, but not all subjects who were proficient in formal operational reasoning exhibited proficiency in systematic or metasystematic reasoning.

The purpose of the present report has been to present the instrument we have developed to assess this higher-order reasoning and to describe the performance of the initial samples of subjects to whom the instrument has been administered. It is, therefore, not appropriate in this report to embark on an extended discussion of the factors or conditions that may govern the development of such higher-order reasoning. To some extent, fruitful speculation in this regard awaits fuller understanding of mechanisms of cognitive development (Kuhn, in press). We should comment, however, that the performance differences between the undergraduate and graduate samples in the present study almost certainly reflect a combined contribution of self-selection and differential experience.
While the "general experience" that comes with increasing chronological age appears to be a sufficient condition for attainment of the earlier stages in Piaget's system, we would not expect it to be a sufficient condition for mastery of the thought operations assessed in the present work. Exposure to and experience with problems that require abstract representational modes of analysis are undoubtedly necessary factors, but just how native ability, education, and experience interact in this regard is a difficult issue to address (see Commons, Richards, & Armon [in press] for additional discussion).

The caveat introduced earlier bears reiteration. It is not necessarily the case, and indeed most unlikely, that all adult thought is of the form investigated here. One of the pressing issues in the study of adult cognitive development, in fact, is to discover the role that formal, logical/deductive reasoning plays in the real-world thought that occurs in adulthood (Gilligan & Murphy 1979, Kuhn et al., in press, Labouvie-Vief 1982). Nevertheless, within the realm of logical hypothetical/deductive thought studied by Piaget, levels of reasoning beyond Piaget's formal operations in our view must be of the general form of third-order and fourth-order operations that we have outlined. It is too restrictive to say that systematic and metasystematic operations are limited to the domains of mathematics and science. Numerous other disciplines, such as literature, history, or anthropology, entail the evaluation of systems within a multisystem framework. It may be the case, however, that systematic and metasystematic reasoning is limited to the domain of formal, abstract, as opposed to everyday, thought, in contrast to Piaget's formal operations which are discernible in everyday thinking. Clearly, a good deal of further work will be required to establish the variety of forms that the systematic and metasystematic reasoning identified in the task presented here may take.

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