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# 3 HOW REINFORCEMENT DENSITY IS DISCRIMINATED AND SCALED

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Understanding how reinforcement schedules are discriminated and scaled should help explain behavior occurring in two different situations. The first of these is where some property of a schedule sample acts as a cue for future behavior, either in the laboratory or in the field (Kamil, Peters, and Lindstrom, 1981; Williams, 1981). As one important example, the effective value of a series of reinforcers may determine upcoming decisions of the subject—for instance, in a foraging situation where the relative density of reinforcement for foraging in a patch controls the choice to stay in that patch or to shift to another. In the second situation, understanding the perceived or scaled value of schedule samples should be useful in explaining the strengthening effects of reinforcement schedules on operant behavior. This is likely because of the high degree of correlation between

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perceived value and choice, as shown by Commons and Ducheny (1981). For example, in chained schedules of reinforcement, the perceived value of the reinforcers in the terminal link may be predictive of behavior in the initial link, whose only reinforcement is the occurrence of the terminal link.

The primary purpose of the present studies is to explore the processes by which pigeons discriminate and scale reinforcement density. The subjects obtained occasional reinforcers on schedule samples for center-key pecks ( $R_C$ ). These schedule samples then served as stimuli to be discriminated. After obtaining the sample, a pigeon indicated whether the sample was from a rich or a lean schedule by a peck to either the left or the right key. In previous work using this method, Commons (1979) found that on the average, pigeons matched probability of a choice, indicating which sample had occurred, to the relative expected payoff for making that choice. They did not maximize payoff by exclusively choosing the appropriate response alternative, as an ideal observer would. However, departures from ideal observer performance may depend on the temporal pattern of reinforcers within the sample. Samples could have reinforcement occurring during any of four sequentially presented 3-second cycles. Therefore, a single reinforcer might occur temporally just before choice or temporally remote from choice. A sample with a single reinforcer occurring right before choice produced more left-key peck choices (the peck indicating that the sample came from the rich schedule) than the sample with the single reinforcer 9 seconds earlier. Choices after samples when the time of reinforcement was either just before or remote did not follow the matching law, whereas the average decision across samples did. Similarly, a sample with a single omitted reinforcer (having three rather than four possible reinforcers) placed immediately before choice produced fewer left-key pecks than a sample with the single omitted reinforcer placed 9 seconds before choice. The change was less for the single missing reinforcer, suggesting that the presence of reinforcement was more salient than its absence.

These previous findings suggested that forgetting might account for the measured discriminability and perceived value of the samples from the two schedules, where forgetting is defined as a decrement in control by a prior event over a future choice as a function of time. The question now is, How might one conceive of a detection and scaling mechanism that is reasonably sensitive to average reinforce-

ment density, but is sent awry by changes, due to forgetting, in the temporal distribution of events within a sample?

To analyze the detection and scaling process, the effect of changing the amount of time between reinforcement opportunities in a schedule sample and the length of the sample were studied in order to learn how decision rules and isosensitivity functions (see Nevin, Chapter 1 for definitions of these two terms) depend on forgetting within a sample in a density of reinforcement discrimination situation. Three experimental manipulations were used to answer these questions. In two of these, cycle length in the four-cycle sample was changed. In the third, the number of cycles was changed, while base (standard) cycle length was kept constant. Cycle length was changed in two ways—either to produce a change to a new cycle length for an extended number of sessions (stable) or to produce a momentary change on a small set of trials within a session (probe).

It is proposed that the determination of perceived density is at least a two-step process. First, summative trace decay theory (Wickelgren, 1974) should account for the relative contribution of each reinforcer within a schedule sample. Summative trace decay theory is modified here and is called the relative-time-weighted sum theory, suggesting that the value of each reinforcer is weighted by how it is discounted, how well it is remembered (Wickelgren, 1974), and that the weighted values are summed. It would predict that as cycle length is increased, whether in a stable or a temporary fashion, the perceived value contributed by each reinforcer should decrease. Second, a set of theories, here referred to as base rate or base time theory, suggests that steady versus momentary changes in cycle length have different effects on perceived density of a sample. One version suggests that a time base and time window are established and that perceived density of the sample reinforcement rate is found relative to those bases. For instance, doubling the cycle length on a probe trial halves the actual rate of reinforcement, which may cause the perceived density to be scaled as half its value on standard or base cycle length trials. This also would leave half the possible events occurring outside the base time window, thus lowering the perceived value further. The effects of adding more events by adding more cycles required explicit examination.

By increasing the number of cycles in a sample while keeping maximally obtainable discriminability constant and comparing the results to those obtained with an equivalently long schedule sample, it is

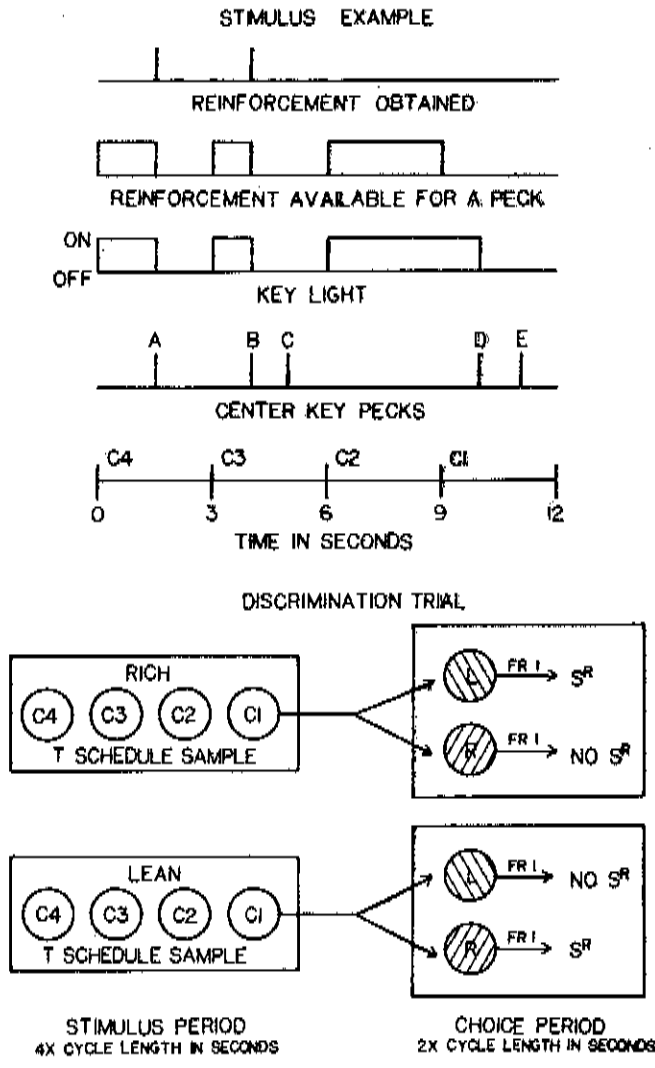
possible to examine the relative contribution of these two sources of discounting information within the sample. First, discounting in the form of forgetting occurs because time between reinforcement opportunities increases and the corresponding number of unprogrammed events increases. Second, as the number of cycles presented within the stimuli to be discriminated increases, the information-processing capacity of the organism is taxed, interference results, and forgetting occurs (D'Amato, 1973; Grant and Roberts, 1973). In either case, both signal detection measures of discriminability and sensitivity ( $d'$ ) and measures of bias ( $p(L)$ ) indicating that the rich schedule has been presented, should decrease, but for different reasons. In Experiment I the effect of changing cycle length is explored.

### EXPERIMENT I: PROCEDURE

Birds were run in one 256-trial session per day. Trials consisted of a stimulus period followed by a choice period, as shown in Figure 3-1. The present task is similar to one in which the subject identifies whether a randomly chosen urn is the rich one that has three reinforcer balls and one nonreinforcer ball or whether the urn is the lean one that has one reinforcer ball and three nonreinforcer balls. On each trial there are four draws from that trial's urn. After a ball is drawn, it is put back into the urn. After the four draws from the urn presented on that trial, the subject indicates whether the sample came from the rich or the lean urn.

Here, the stimuli to be discriminated were rich,  $p(S^{R+}|R_C) = 0.75$ , and lean,  $p(S^{R+}|R_C) = 0.25$ , schedules for center-key pecks,  $R_C$  presented as modified T schedules (Commons, 1979; Schoenfeld and Cole, 1972; Schoenfeld, Cumming, and Hearst, 1956; Weissman, 1961). On each trial, a sample from one of the two T schedules was presented during the stimulus period. Each one of these sixteen sub-stimulus samples consisted of four equal duration cycles. On each cycle, a center-key peck was ( $v_i = 1$ ) or was not ( $v_i = 0$ ) reinforced. The cycles,  $c_i$ , were numbered so that  $C1$  is the cycle at the end of the stimulus period and right before the choice period and  $C4$  is at the beginning of the stimulus period and the furthest from the choice. An unlikely example of what might happen during a presentation of a 1110 substimulus sample is shown in the upper portion of Figure 3-1. As is shown, each cycle began with the illumination of

Figure 3-1. The top portion shows a state diagram for a sample from a 3-second cycle schedule. It illustrates what may happen if center-key pecks occur and do not occur when reinforcement has been programmed or not. It is for illustrative purposes only and would be very unusual because there is a cycle without a peck occurring in it. The bottom portion shows the contingencies during an entire trial. The stimulus period contains substimuli of the form shown in the top portion. The choice period immediately follows the stimulus period. There is no intertrial interval.



the center key. The first center-key peck darkened the key and was reinforced with the same probability as the rest of the cycles on that trial,  $p(S^{R+}|R_C)$  being either 0.75 or 0.25. No other center-key pecks during that cycle were reinforced, although they occurred.

At the onset of the choice period the side keys were illuminated and the center key stayed dark or was darkened in those rare cases where no key peck occurred in the last cycle of the substimulus. The choice period duration was always twice the standard or base length cycle, as shown in the lower portion of Figure 3-1. The first side-key peck, whether correct or not, darkened both keys, and no further pecks were counted. If a substimulus sampled from the rich schedule had been presented on the center key, the first left-key peck was reinforced (a hit or left correct); a right was not reinforced (a miss or right error). If a substimulus from the lean schedule had been presented on the center key, the first right-key peck was reinforced (a correct rejection or right correct); a left was not reinforced (a false alarm or left error).

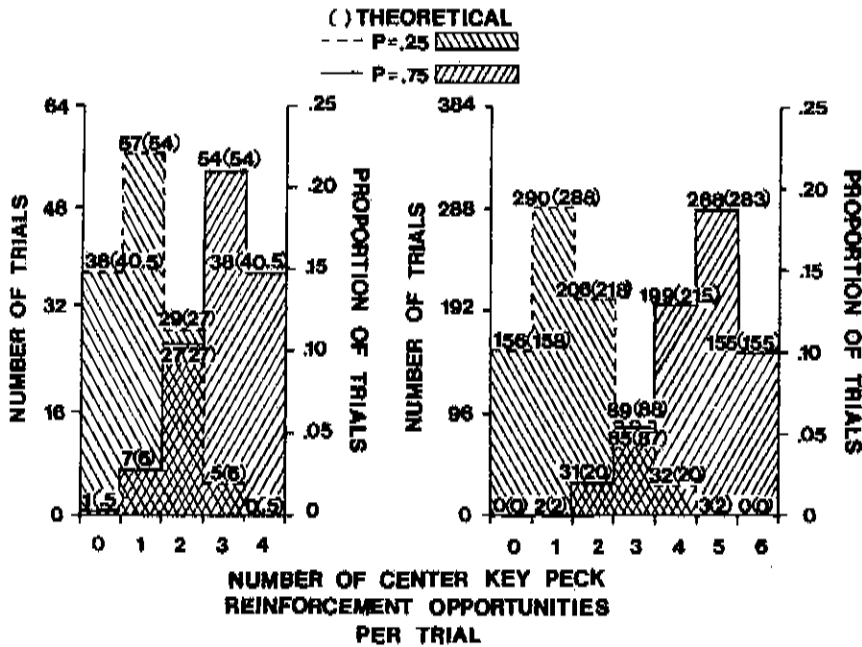
Three different standard cycle lengths were used—2 seconds, 3 seconds, and 4 seconds ( $T = 2, 3, 4$ ). Each was run until the birds stabilized. In daily sessions the standard (base) cycle length was of standard length (i.e.,  $T$  multiplied by 1) on 224 trials, doubled ( $T$  multiplied by 2) on 16 probe trials, or tripled ( $T$  multiplied by 3) on another 16 probe trials. The position of the probe trials within a session was randomly distributed. The frequency of the occurrence of the possible substimuli for standard and probe trials is shown in the left panel of Figure 3-2. The modal density of the rich schedule is 3; the modal density of the lean schedule is 1.

### Stimulus Description

There are four levels of description of the stimuli.

1. On a "macro" level, each probabilistic reinforcement schedule, either the rich ( $p = 0.75$ ) or the lean ( $p = 0.25$ ), is viewed as a single "substimulus," here called  $S_{\text{rich}}$  and  $S_{\text{lean}}$ , respectively; sampling considerations are of no concern. These stimuli are equivalent to cued random interval (RI) or variable interval (VI) schedules with a  $T$ -second limited hold and average intervals equal to  $4/3T$  seconds and  $4T$  seconds.

Figure 3-2. The distribution of reinforcement for the first center-key peck ( $S^{R^+}|R_C$ ) for four- (the left panel) and six-cycle stimuli (the right panel) are shown. The number of center-key pecks and proportion of trials on which reinforcement occurs in a sample is shown for the rich (positive slope hatching) and lean (negative slope hatching) schedules. The respective  $p(S^{R^+}|R_C)$  are 0.75 and 0.25 for the four-cycle stimuli and 0.767 and 0.233 for the six-cycle stimuli.



- At the "molar" level, all substimulus samples with the same number of reinforcements are viewed as the same "stimulus." Each substimulus,  $S_n$ , has a reinforcement density,  $D(S_n)$ , equal to  $N_d$ , the number of center-key peck reinforcement opportunities over the four cycles in the substimulus. Density, the defining characteristic of a stimulus at this level of analysis, ranges from zero to four reinforcers per substimulus, giving rise to five such "stimuli," designated  $S_{N_d}$ . This level of analysis distinguishes between the number of reinforcements within a sample stimulus but not between the particular  $T$ -second cycles within that  $4T$ -second stimulus period on which those reinforcements are programmed.

3. At the "molecular" level, the definition of a "stimulus" involves the pattern of reinforcement in a substimulus. Each substimulus is represented as a four-digit binary number. A number such as 0001 indicates a sample with three cycles without reinforcement opportunities followed by one cycle with an opportunity. The substimuli,  $S_n$ , are numbered from 0000 to 1111. The leftmost digit represents the cycle furthest in time from choice, and the rightmost represents the cycle immediately before choice.
4. At the "micro" level, the definition of a "stimulus" depends on whether or not there is a reinforcement opportunity on a particular cycle,  $T$ -seconds preceding choice, irrespective of what is programmed for its neighbors.

There are sixteen possible combinations in a 4-cycle substimulus. The probability that a particular set of four events will occur was obtained by expanding the binomial  $(p+q)^4$  with  $p = 0.75$  or  $0.25$  and  $q = 1 - p$ . Actual presentation frequencies deviated somewhat from the expected frequencies because of sampling.

A molecular substimulus, even 0000 or 1111, could occur with either schedule stimulus. However, the greater the number of cycles having a reinforcement opportunity, the greater the likelihood that the rich schedule was in effect. Therefore the lean (or rich) schedule was more likely to be in effect when 0 or 1 (or 3 or 4) reinforcement opportunities were presented on a trial (respectively). Substimuli with 2 reinforcement opportunities occurred about equally often given either schedule. Birds were run until their performances stabilized. The data reported here were collected in the five sessions following stabilization.

### Results

The results of the present study encompass several levels of description along two stimulus dimensions, one reflecting the number and distribution of reinforcers in a sample substimulus and the other, cycle length, which determines the temporal distance of reinforcers to choice. For the reinforcement dimension, at the macro level, the sensitivity of the birds to the difference between two different reinforcement distributions will be described for the nine possible combinations of cycle lengths. At the molar, molecular, and micro levels, the decision rules, which describe the relationships between choice



and particular substimulus parameters such as number of reinforcements in a substimulus and their relative distance from the choice point, can be studied in such a way as to explain the birds' sensitivity to and perceived value of the substimulus.

For the second dimension, cycle length, two operations were used to generate the temporal distance of a reinforcer in a given substimulus from choice. Each can be used to examine the effects of cycle length on perceived value. One operation established one of three standard or base cycle lengths over a large number of sessions (stable change). The second operation either doubled or tripled the standard cycle length on probe trials (momentary change). Presentation of results will proceed from most aggregated to least aggregated, along both dimensions. Sensitivity will be discussed before perceived density, although the former may depend on the latter.

First, the effects of lengthening cycles on  $d'$  and  $p(L)$  are examined at the macro level. The leftmost panels in Figure 3-3 show the  $p(\text{Hit})$  and  $p(\text{False Alarm})$  coordinates for each value of overall cycle length, irrespective of how the cycle length was obtained (i.e., from a standard or a probe trial). These are superimposed on receiver-operating-characteristic (ROC) curves, which were calculated for a number of levels of sensitivity, according to the continuous binomial method detailed in Commons (1979). There are two values associated with each point. These are  $d'$ , which reflects sensitivity to the difference between the density of the rich and lean stimuli, and  $p(L)$ , the probability of a left-key peck, which is the tendency to indicate that the substimulus came from the rich schedule. The value,  $p(L)$ , is a measure of perceived reinforcement density (bias). From an examination of the position of the points relative to the ROC curves for sensitivity and to the negative diagonal for bias, one can see the effects of increasing cycle length or of the number of cycles on sensitivity and perceived value.

Both  $d'$  and  $p(L)$  decreased as cycle length increased. The decrease in  $d'$  was indicated by the points falling nearer the positive diagonal, the isosensitivity curve that reflects no sensitivity. The decrease in perceived density was indicated by points falling closer to the  $x$  axis and farther from the negative diagonal. On probe trials, both sensitivity and perceived density should decrease together, because assigning a lower density to a sample than it has decreases the hit rate (see Nevin, Chapter 1, for a definition) without decreasing the false alarm rate.

Figure 3-3. Column 1.: Isosensitivity curves for four birds (top four panels) discriminating four-cycle samples and three birds discriminating six-cycle samples (bottom panels) are shown. The probability of a hit,  $p(\text{Hit})$ , is equal to the number of correct left-key pecks divided by the number of possible correct left-key pecks. The probability of a false alarm,  $p(\text{False Alarm})$ , is equal to the number of incorrect left-key pecks divided by the number of possible correct right-key pecks. The solid lines show isosensitivity curves for eight values of  $d'$ , a measure of sensitivity. The top curve would be obtained if the subject followed an ideal decision rule derived from the continuous binomial distributions in Figure 3-2. The closer points fall to the 0,0 corner the smaller the perceived density of the substimulus. Throughout the rest of the figures a common symbol code is used. Circles, triangles, and squares represent 2-, 3-, and 4-second base (standard) cycle length. The open, half-filled, and completely filled symbols represent trials with standard cycle length (open) and probe trials obtained by doubling (half-filled) and tripling (completely filled) standard cycle lengths. The second, third, and fourth columns plot  $d'$ , a sensitivity measure found from the inverse to the normal probability distribution as an approximation to the binomial. Column 2 shows  $d'$  versus cycle length irrespective of how obtained; whereas Column 3 shows  $d'$  as a function of the base, doubling of base, and tripling of base cycle length on probe trials; and Column 4, as a function of base cycle length. Best fit lines indicate the other parameter.

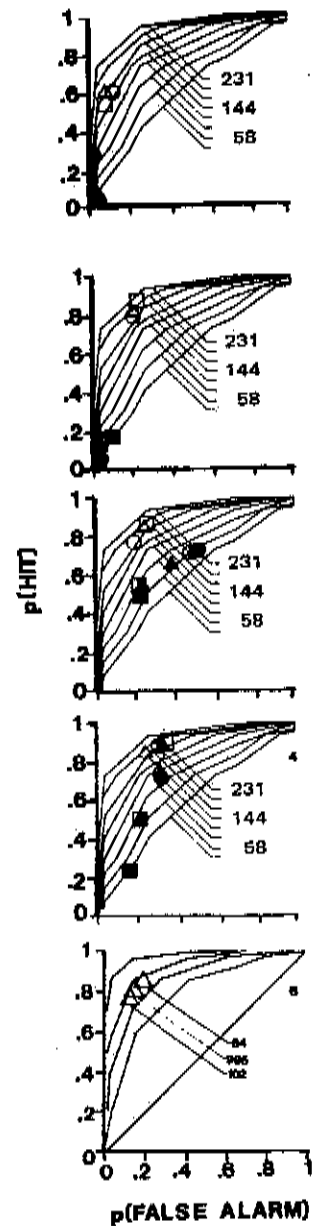
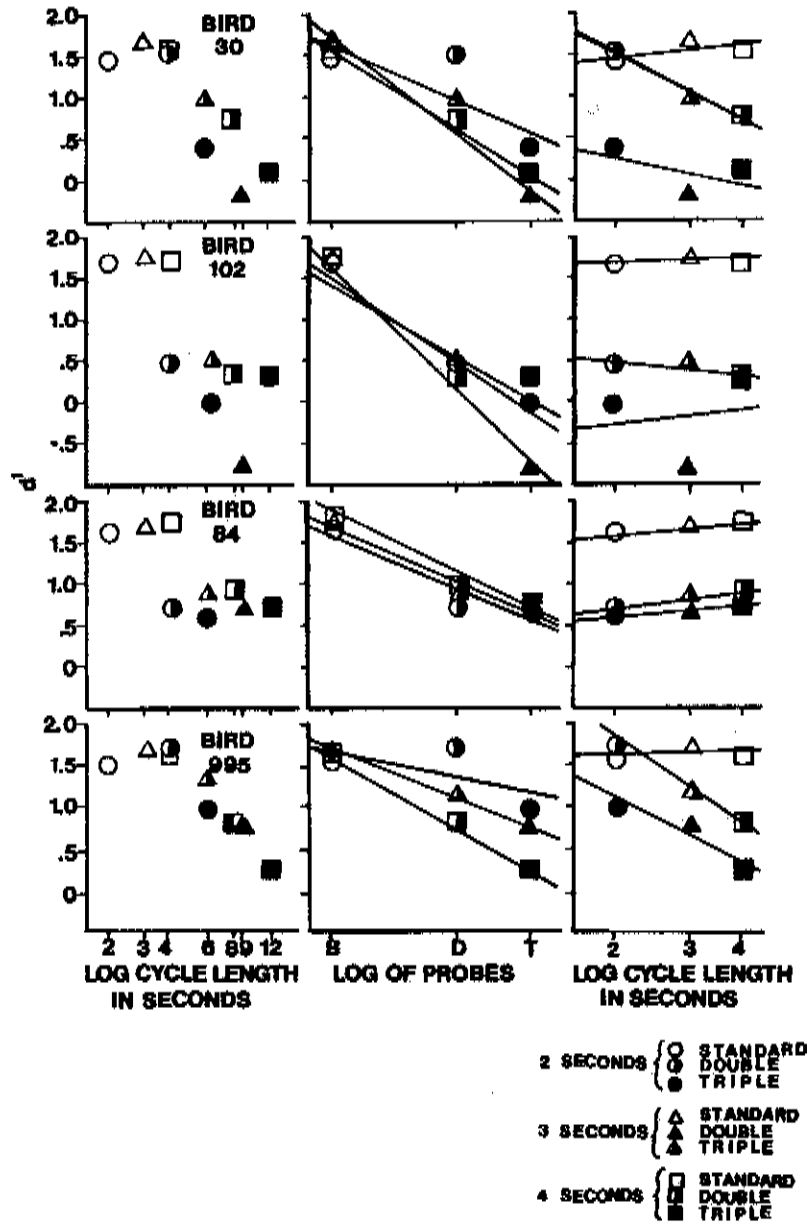


Figure 3-6. continued



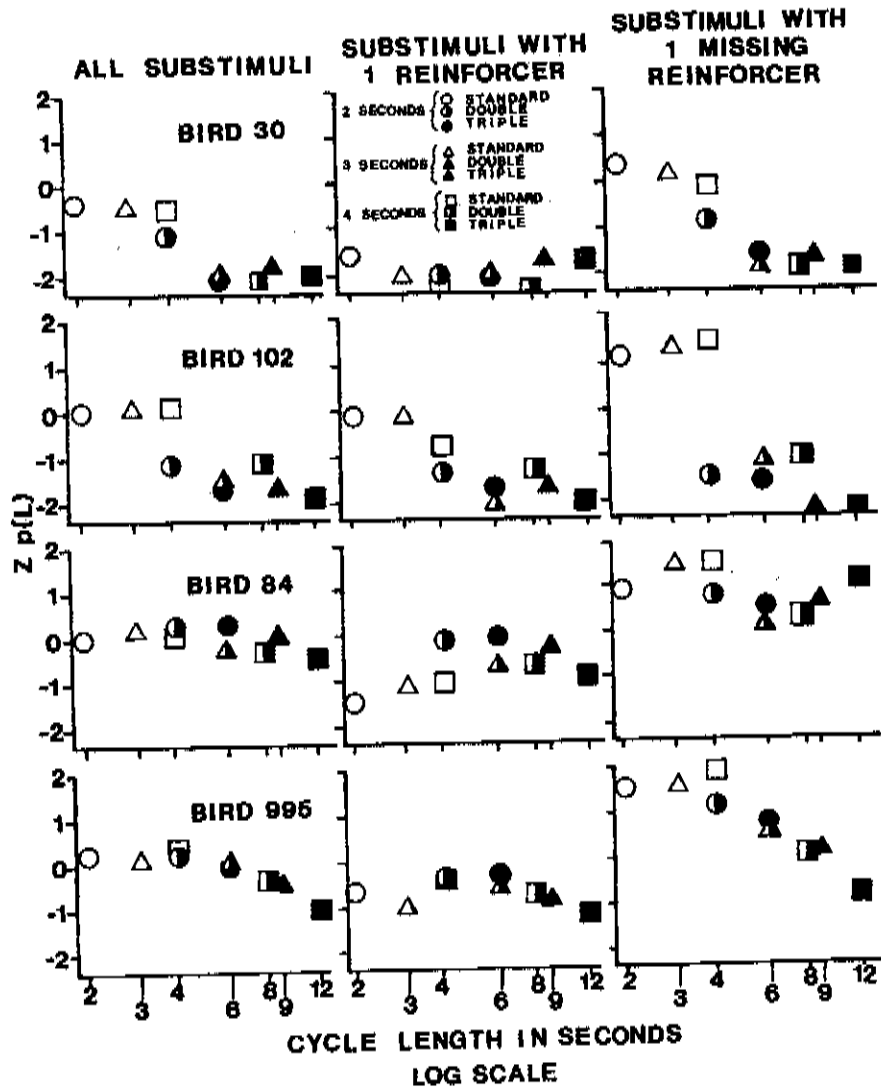
The general decrease in  $d'$  as a function of increasing cycle length is more clearly seen in the second column of panels in Figure 3-3. The  $d'$  values were found by assuming the normal approximation to the binomial rather than the continuous binomial model reported in the leftmost panel of Figure 3-3. Here, the sensitivity,  $d'$ , is plotted against log cycle length, the time dimension being logged to make the relation more linear. Bias is examined separately in Figure 3-4. For Birds 30, 102, and 995,  $d'$  was slightly higher at the 3-second cycle length than at either the 2- or 4-second base cycle lengths. From our observations, the birds had trouble pecking the center key and picking up the programmed reinforcer and then repositioning themselves in front of the center key all within two seconds. The cycle lengths greater than 4 seconds were obtained by either doubling or tripling different standard (base) cycle lengths. When disregarding how a given cycle length was obtained, this graph clearly shows that as cycle length increased beyond 4 seconds, there were large decreases in  $d'$  for Birds 30 and 995, a less uniform but large drop for 102, and no consistent pattern for Bird 84.

The panels in the third and fourth columns of Figure 3-3 segregate the information from the second column so that the effects of the two ways of producing different cycle lengths may be clearly seen. Sensitivity to the differences between the two schedules did not decrease in a simple fashion because the two mechanisms for lengthening cycles did not produce equal effects. Momentary increases in cycle length on probe trials produced the largest decrease in sensitivity, as shown by the negative slopes in the panels of the third column and the different heights of lines in the panels of the fourth column. Generally, the longer standard cycle lengths, when doubled and tripled, produce lower sensitivities, especially for Birds 30 and 995, indicating an interaction between probe ratio and standard cycle length. This can be seen by the generally descending order of sensitivity from the logged standard (base) cycle lengths to the doubled and tripled probes in column 3 and by the difference in the slope of the lines in column 4. One critical pair of points in column 4 occurs for the 6-second cycle length, because it can be obtained by either doubling the 3-second standard or tripling the 2-second standard. The former point is a half-filled triangle, and the latter is a fully shaded circle. The doubling of 3 seconds produces less of a sensitivity loss than the tripling of 2 seconds.

The leftmost panel of Figure 3-4 replots the perceived value (bias) of the stimuli as a function of cycle length. This information was previously shown in the leftmost panel of Figure 3-3, but in Figure 3-4 it is plotted separately from sensitivity. Calling bias, which is produced by a stimulus, perceived density can be justified as follows: If  $p(L)$ , the tendency to indicate that a sample came from the rich schedule, equals 1, when a sample from the rich schedule has been presented, then the bird has responded correctly and is reinforced. If  $p(L)$  equals 0 when a sample from the lean schedule has been presented, again the bird has responded correctly and is reinforced. Hence, the value of  $p(L)$  indicates the perceived density of a schedule sample. The tendency to say that the sample came from the rich schedule,  $p(L)$ , was transformed into the corresponding  $z$  value by the probit, or inverse probability, transformation in order to produce more linear functions. The resulting quantity,  $z_{p(L)}$ , is the perceived density of a schedule sample. The  $z_{p(L)}$  was plotted against the log of cycle duration. The first panels show the effect of cycle length at the macro level. The perceived value of a substimulus generally rose or stayed nearly the same as cycle length increased from 2 seconds to 3, either decreased or stayed the same for cycle length from 3 to 4 seconds, and decreased in a not entirely linear fashion for cycle length above 4 seconds.

At the molar level, the perceived value of substimuli with one reinforcer is shown in the middle panels and that of substimuli with three reinforcers in the rightmost panels. While performance characterized at the macro level reflected the birds' scaling of the perceived value of all the substimuli together, the molar level analysis focuses on their responses to substimuli with the same reinforcement densities. The perceived values changed in a reasonable fashion for substimuli with one reinforcer or one missing reinforcer. The panels in the middle and right columns of Figure 3-4 indicate that most of the birds show maximum perceived value at around 4 to 6 seconds. There was no consistent change in perceived value as a function of standard cycle length alone. However, when combined with doubling and tripling on probe trials, there is generally a decrease in perceived value as a function of overall cycle length (i.e., the half-filled and filled points generally fall below the unfilled points). The form of the function is not due to different density substimuli being treated differently, as can be seen from the similarity of patterns in the panels in columns 2 and 3, whereas the height of the function is.

Figure 3-4. Perceived density,  $z_p(L)$ , plotted against cycle length in log seconds. The left-hand column shows the effect of cycle length on the perceived density of all stimuli considered together. This is the macro level relation. At the molar level, Columns 2 and 3 show the relation between perceived density and cycle length for stimuli with one reinforcer and stimuli with three reinforcers (one missing reinforcer).



In the following account of the foregoing performance at both the macro and the molar level, it is predicted that perceived density (the inverse probability transform of the probability of a left-key peck) is a linear function of actual density. First, because of the way that substimuli are distributed between the rich and lean schedules (Commons, 1979), the relative expected payoff,  $Rel EP(L|S_{N_d})$ , was an ogival function of reinforcement density. The relative expected payoff for a left-key peck at a given density substimulus,  $D(S_{N_d}) = N_d$ , is just the probability of a left-key peck being correct,  $LC$ , at that density,  $N_d$ , when amount of reinforcement for each correct choice is equal.

$$\begin{aligned} Rel EP(L|S_{N_d}) &= \frac{p(LC|S_{N_d})}{p(LC|S_{N_d}) + p(RC|S_{N_d})} \\ &\text{but } p(RC|S_{N_d}) = 1 - p(LC|S_{N_d}) \\ &= \frac{p(LC|S_{N_d})}{p(LC|S_{N_d}) + (1 - p(LC|S_{N_d}))} \\ &= p(LC|S_{N_d}) \end{aligned}$$

The counterpart of  $Rel EP(L|S_{N_d})$  or  $p(LC|S_{N_d})$  in free operant schedules is programmed reinforcement rate. This fact will be used to suggest that the mechanism that gives rise to the matching result will generalize to the more usual free operant schedules.

Second, applying a probit transformation (the inverse of the standard normal cumulative distribution function) to the function that relates relative expected payoff to density turns it into a straight line with slope 1.084, intercept -2.110, and an  $r^2 = .996$ ;  $z_{p(LC|S_{N_d})} = -2.1(1 - N_d/2)$ . This linearity results from the goodness of the normal approximation to the binomial, which was used to generate the stimuli. In Figure 3-5, the independent variable can be expressed in two ways—actual reinforcement density of the substimulus or probability transform of relative expected payoff for indicating that a substimulus had high density. If perceived density is a linear function of actual density, it is also a linear function of relative expected payoff in  $z$  form and vice versa. As a consequence, the matching relation

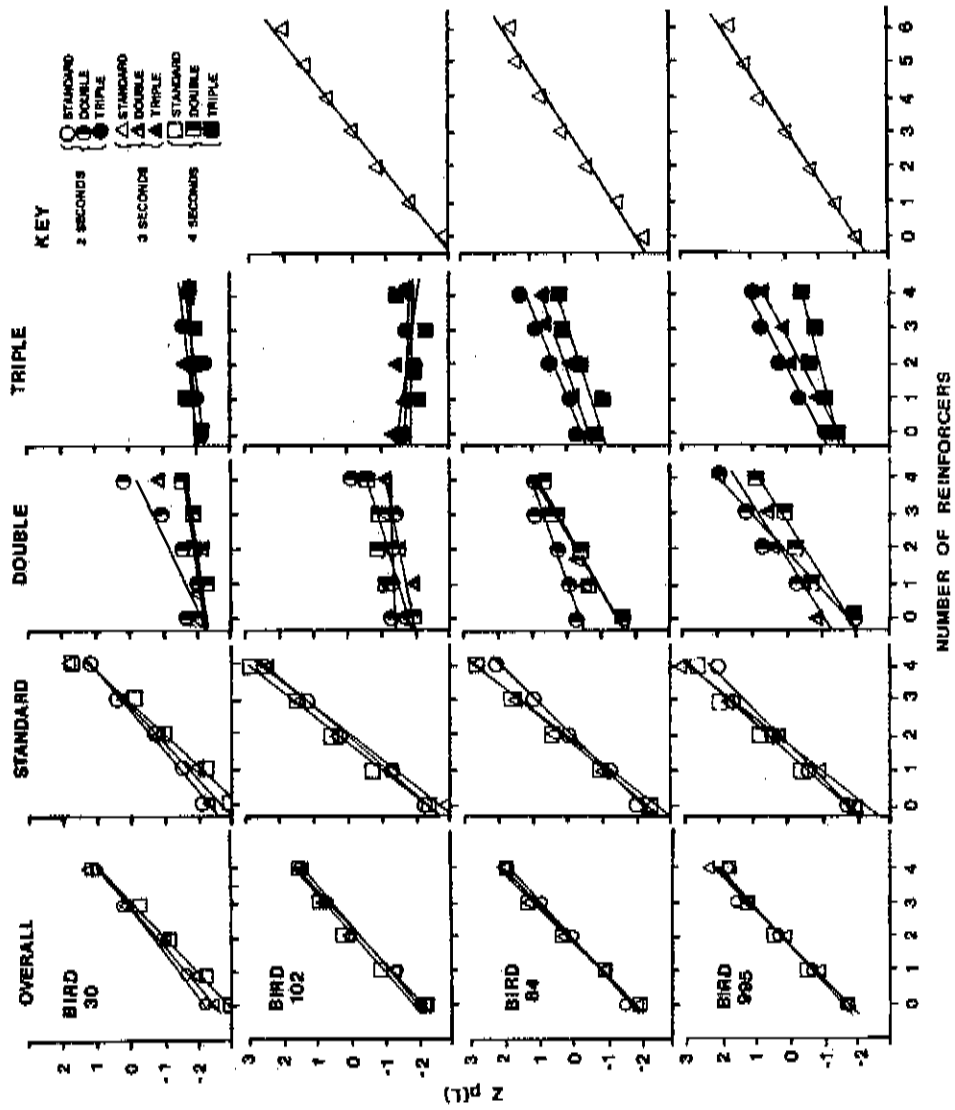


Figure 3-5. Perceived density,  $Z_p(L)$ , shown as a function of actual density (the number of reinforcers in a substimulus). This molar relation is shown for all trials together (Column 1), for trials with standard cycle length (Column 2), for probe trials where the standard length was doubled (Column 3) and tripled (Column 4), and for six cycles at standard length (Column 5).



holds (i.e., perceived density is equal to the probit transform of relative expected payoff) if perceived density is found to be a linear function of actual density. Since matching was found to hold for similar stimuli, but without probe trials being included in the sessions (Commons, 1979), it would be predicted to hold here.

Figure 3-5 shows that matching at the molar level holds when the entire session is considered. The relations shown within the figure rejected a number of ways that pigeons might scale reinforcement density in samples, while supporting others. The decision rules for each bird at the molar level are graphed with respect to different parameters. The decision rule examined here is the psychophysical relation between the perceived reinforcement density,  $z_{p(L)}$ , and the actual reinforcement density—with cycle length, obtained by two different manipulations, being the parameter. The leftmost column of panels in Figure 3-5 shows the molar relation between the perceived density,  $z_{\text{average } p(L|S_n, D(S_n) = N_d)}$ , and actual stimulus density,  $N_d$ , for three different cycle lengths for the combination of probe and standard trials. The parameter is cycle length. The points were well described by the regression lines fit by the median method (Mosteller and Tukcy, 1977), with the  $r^2$  values ranging from .98 to .99, as shown in Table 3-1.

That the birds linearly scale reinforcement density in the schedule samples of the duration studied here implies that the matching law holds at the molar level, since linear density scaling implies matching, as discussed above. The matching law and linear density scaling were further supported by a number of related facts. In  $z$  form, the mean perceived density was 0 for the mean actual density of 2 (density 2 stimuli), as it should have been: This means that these stimuli were seen as coming equally from either distribution. The perceived values were symmetrically distributed about 0 with perceived density ranging from a value of -2.2 for density 0 stimuli to +2.2 for density 4 stimuli.

For the standard cycle lengths alone in Figure 3-5, functions relating perceived density to number of reinforcers were steeper than those for the combined trials. This suggests if matching is fundamental, then the birds compensate for the inclusion of decrement-producing probe trials by overvaluing stimuli on standard trials. The  $r^2$  values for the best fit lines range from .98 to .99.

One can see how density was scaled by examining the role played by two ways of lengthening cycles in producing the decrement in

Table 3-1. Decision Rules at the Molar Level.

	Slope $Z P(L)/Density$ ( $Z EP(L)/Density$ )	Intercept $Z P(L)$	$R^2$	Ratio of Slopes of Standard Cycle Length to Given Slopes	Model I Predicted Ratios of Slopes
Predicted by matching	(1.084)	-2.11	.992		
Overall					
2 sec	0.919	-1.941	.984	1.464	
3 sec	0.979	-2.04	.972	1.230	
4 sec	0.953	-1.917	.975	1.282	
Standard					
2 sec	1.054	-2.16	.992	1.000	
3 sec	1.204	-2.37	.993	1.000	1
4 sec	1.222	-2.22	.993	1.000	1
Double					
2 sec	0.485	-1.462	.556	2.17	2
3 sec	0.421	-1.735	.457	2.86	2
4 sec	0.451	-1.741	.659	2.71	2
Triple					
2 sec	0.234	-1.286	.106	4.50	3
3 sec	0.226	-1.402	.155	5.34	3
4 sec	0.112	-1.557	.087	10.91	3
Six cycle standard					
3 sec	0.690	-2.21	.976		

sensitivity. Again, the two ways were to change cycle length on selected probe trials by doubling or tripling the standard cycle length or to change the standard cycle length over an extended period of time. Any model that suggests that the birds responded simply on the basis of number of reinforcers, independently of context or time, must predict that momentary changes in cycle length should have no effect on perceived value. Any model that proposes that the birds responded on the basis of the relative time between reinforcers or rate of reinforcement must predict that perceived value should be inversely proportional to momentary cycle length. If the standard cycle is doubled on a series of trials, the perceived value should be halved. Likewise, the ratio of the slopes of functions relating perceived density to actual density should be halved. A third alternative is that the birds perceived something like a weighted average rate or weighted average time between reinforcers. This model should include a term for the interaction between standard cycle length and ratio of probe length to standard.

How well these various models are supported by the data is seen in all the panels in columns 3 and 4 of Figure 3-5. The perceived sub-stimulus density decreased more with the insertion of probes than with increases in standard cycle length, as is shown by the flattening of the slopes for doubles and triples. The probes caused even greater decrements as the standard cycle length increased.

Any model that depends on number of reinforcers alone is rejected by the fact that there were changes in slope with increased cycle lengths, indicating that time was indeed important. The standard cycle slopes were 1.0, 1.2, and 1.2 at 2, 3, and 4 seconds, respectively; doubling slopes were .49, .42, and .41 (clearly lower); and the tripling slopes were lower still— .24, .21, and .15 (as shown in Table 3-1).

Doubling and tripling standard cycle lengths decreased the perceived density more than predicted by time- or rate-averaging models. The ratio of the slopes, double to standard and triple to standard, would be 2:1 and 3:1, respectively, if the weighted average rate or weighted average time model were true in its simplest form. As shown in Table 3-1, the ratios of the slopes for the average of the four birds are 2.0, 2.9, and 2.9 for 2-, 3-, and 4-second cycle lengths for doubling and 4.2, 5.7, and 8.0 for tripling. There may be an interaction between standard cycle length and probe value: At least for Birds 84 and 995, tripling the 4-second standard had a larger dec-

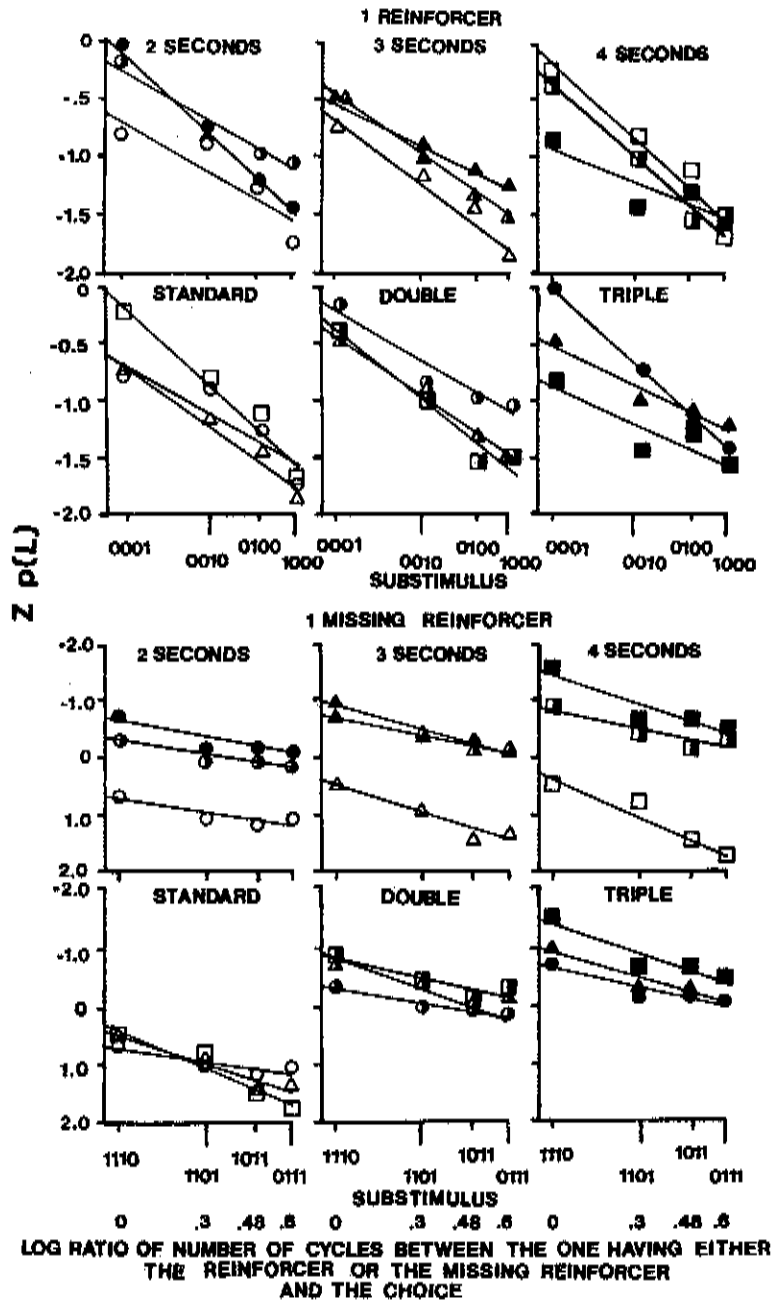
Figure 3-6. Perceived density (or its negative),  $z_{p(L)}$  or  $-z_{p(L)}$ , shown as a function of the log of the ratio of the number of cycles before choice of either the cycle with the single reinforcer or the one missing reinforcers. For substimuli with one reinforcer, the top row shows this as a function of the various cycle lengths. The second row shows this for standard trials and probe trials on which standard cycle lengths were doubled and tripled. The bottom two rows show this relation for substimuli with one missing reinforcer (three reinforcers). The  $y$  axis is inverted for the bottom two rows so that a direct comparison may be made between the top two rows and the bottom two rows, with row 1 and 3 corresponding and row 2 and 4 corresponding.

remental effect on the slope than tripling the 2-second standard. While these slope changes are in the right direction, they clearly deviate from ratios predicted by time or rate averaging, especially as standard cycle length increases. This is not surprising. The birds do not compensate for the fact that the probe substimuli start much earlier than the standard. The decrement in perceived density is greater than would be the case if those earlier events in the substimuli were not there.

If average reinforcement rate determined perceived density, reinforcers occurring more cycles away from choice would not be weighted less. However, as weights decrease farther from choice, two versions of weighting have to be considered—one that hypothesizes exponential decay (i.e., relative time alone is important) and a second that hypothesizes a limited event window or short-term working memory (i.e., number of intervening events is important).

To see whether this decrease occurs, the effect of a single reinforcer (or single missing reinforcer) occurring farther from choice is examined at the molecular level. Averages of four birds' perceived density are shown in Figure 3-6 as a function of how far the choice is from either a single reinforcement opportunity or a single missing reinforcement opportunity, both here called critical events. Averages of the four birds' performances were used because, at least for the probes that doubled and tripled standard cycle length for each particular substimulus, the frequencies of those substimuli were small, resulting in noise. The data for substimuli with one reinforcer are shown in the top two rows of panels of Figure 3-6, and those for one missing reinforcer in the bottom two rows. A point in Figure 3-6 at density 1 is the average across birds for density 1 substimuli of the same cycle length. Likewise, a point at density 3 is an average

(Figure 3-6)



across birds of density 3 (one missing reinforcer) substimuli of the same length.

The data are again graphed in two ways. The standard cycle length parameters are graphed as circle (2 seconds), triangle (3 seconds), and square (4 seconds). How much a particular symbol is filled depends on whether it represents a standard (unfilled), doubled (half-filled), or tripled (completely filled) cycle. Some general properties emerge. There is a larger decrement for one reinforcer than for the one missing reinforcer, as shown by the slopes for the former being greater than for the latter. This suggests that the diminution of control is greater for an event rather than for a missing event. Other effects are not as orderly, perhaps because some of the events are occurring very far from choice in time, and  $z_{p(L)}$  values are close to either  $-2$  or  $+2$ .

#### A Micro-Molecular Model that Accounts for Perceived Density of a Substimulus

I suggest that perceived density of a substimulus,  $D^*(S_n) = z_{p(L|S_n)}^*$ , is the sum of the weighted values of reinforcers that occurred in the substimulus. On each cycle,  $c_i$ , a reinforcer in a substimulus may occur ( $v_i = 1$ ) or not occur ( $v_i = 0$ ). Cycle number,  $c_i$  or  $i$  alone, is counted backwards from choice and has values 1, 2, 3, 4, the value 1 being right before choice and at the end of the stimulus period.

A multiple regression was performed to determine the contribution of a reinforcer on a given cycle. Remember that a substimulus has four such cycles, so that there are four coefficients,  $a_i$  for  $v_i$ , and a constant,  $a_0$ , to be estimated for this molecular model.

$$\begin{aligned} D^*(S_n) &= z_{p(L|S_n)}^* = a_0 + a_1 v_1 + a_2 v_2 + a_3 v_3 + a_4 v_4 \\ &= a_0 + \sum_{i=1}^4 a_i v_i \end{aligned}$$

The weights,  $a_i$ , represent the contribution of a reinforcer on a respective cycle,  $c_i$ . For substimulus,  $S_0 = 0000$ , all the  $v_i$ s are 0, so that  $a_0$  represents the perceived density of a substimulus with no reinforcers. The independent variable was  $v_i$ , whether or not a reinforcement opportunity occurred on a given cycle number; the per-

ceived value of each of the sixteen substimuli at 2-second standard cycle length was the dependent variable. As shown in the second column of Figure 3-5, the other functions were almost identical, so only the analysis of the 2-second data is reported. This analysis of all four birds together yielded a molecular model with five estimated parameters,  $a_0$  through  $a_4$ , -2.16, 1.40, 1.168, 0.900, and 0.7125, and a multiple  $r$  of .91. For individual birds the coefficients were slightly different, and  $r$  values were slightly higher ranging from .96 to .99, since variability between individuals was eliminated. Since linear regression depends on the predictor variables being additive, the fact that so much of the variance was accounted for suggests that substimulus density is the sum across the  $i$  cycles of the contribution of events on each of the  $i$ th cycles. This finding is consistent with Wickelgren's (1974) assumption that the contribution of multiple occurrence of the events to be remembered is additive.

The heart of the matter is to see if Wickelgren's (1974) suggested exponential equation, which represents forgetting, accounts in a more specific fashion for the contribution of the programmed events on a given cycle. While the previous multiple regression reduced the number of parameters necessary to describe the data and showed additivity, using an exponential model should allow a more compact description. The coefficients,  $a_i$ , for each cycle were previously the parameters to be estimated. Since the micro model suggested that each term was exponential, the parameters,  $a_i$ , now become the dependent variables, and the independent variables,  $c_i$ , were used in the expression  $a_i e^{b_i c_i}$  to predict them. The values of  $a = 1.793$  and  $b = -0.229$  so found yielded a correlation between cycle number away from choice and log cycle number weight (from the previous regression) of  $-0.9975$ ,  $r^2 = .995$ . Hence, the model's coefficients predicted perceived value as well as the multiple regression and yet had only three parameters for sixteen substimuli. To further test this model of what happens on a given cycle (micromodel), the predicted coefficients were entered into the multiple regression as multipliers of  $v_i$ . If the micro model worked well, the multiple  $r$  value should be the same as the molecular, in which the coefficients were directly obtained as a best fit ( $r = .912$ ), and the coefficients should be very close to 1. The residuals should not be correlated with cycle number. The obtained  $r$  for this procedure was .912; the coefficients were 0.98, 1.03, 1.00, and 0.99; the constant was  $-0.216$ ; and the residu-

als were uncorrelated. The combined micro-molecular model equation for the perceived density of a substimulus is:

$$D^*(S_n) = z^*_{p(L|S_n)} = -2.16 + \sum_{i=1}^4 v_{c_i} 1.8e^{-.229c_i}$$

This finding on the form of the decay function is similar to Wickelgren's (1974) in that for the short term, events lose impact as a function of the negative exponential of time,  $a_i e^{-b_i T_i}$ . Some comparisons of the present model for pigeons with his model for adult humans should be made. Here, it was the number of cycles before choice (the relative temporal distance away from choice,  $c_i$ ) on which each reinforcement and non-reinforcement opportunity fell that accounted for the decay, instead of time away from choice,  $T_i$ . The latter had no effect on perceived density, as shown for standard length cycles in column 2 of Figure 3-5. Here, because it was how many cycles before a choice that mattered and not simply time before choice, a pure timing or temporal decay interpretation is unwarranted. Each cycle may act like an event. Knowing how many cycles before choice a given event fell tells us how many intervening events there are. Whether or not the form of the decay function is due to interference of intervening events or simply to the passage of time was not answered by Wickelgren. The fact that number of events to be processed may be the critical variable suggests an examination of the effects of number of cycles on perceived density and sensitivity, the next experiment to be described.

## EXPERIMENT II: SIX 3-SECOND CYCLE SAMPLES

### Method and Procedure

The subjects and apparatus for this experiment were identical to those described in Experiment I, except that one less subject was used. The discrimination task was similar to that used in Experiment I, except that the number of cycles was increased from four to six. The stimulus on each trial, then was one of 64 substimuli (of six equal duration cycles) randomly selected from one of two T schedules. The two schedules to be discriminated were, again, two



overlapping distributions (stimuli) of reinforcement opportunities—one with  $p(S^{R+}|R_C) = 0.767$  (the rich schedule) and one with  $p(S^{R+}|R_C) = 0.233$  (the lean schedule). These probabilities were chosen so that the ideal observer's discriminability,  $d'$ , based upon the overlap of the discrete binomial distribution, was the same as in the four-cycle case. The rich and lean binomial frequency distributions are shown in Figure 3-3.

### Results

The  $d'$  values reported in this section were found by assuming a continuous binomial model and were then compared to the  $d'$  values calculated in the same manner for the same birds in the four-cycle case. These  $d'$  values differed slightly from those obtained with the normal approximation to the binomial reported everywhere else in this chapter.

The  $d'$  values shown in Table 3-2 were lower for the six 3-second cycle sample discrimination than for the four 3-second cycle sample discrimination, whether the values were found by assuming a continuous binomial or a normal approximation to the binomial. The discrimination efficiency,  $e$ , was much lower for the six 3-second cycle sample (18-second long sample) than for the stimuli with four 4-second cycles (16-second long sample). Here,  $e$  is the ratio of obtained  $d'$  to maximum possible  $d'$ . This supports the notion that there is much more of a problem with processing more programmed events than with processing extraneous events occurring over time, such as the house light, which was off, or the hopper light, which was on.

The slopes of the six-cycle decision lines were not as steep as the four-cycle ones. The addition of two extra events to the four-cycle samples decreased sensitivity and altered the decision rules more than might be expected if maximizing one alternative as soon as it became more favorable held instead of matching. If a maximizing account were closer to the truth, the birds might have treated both a density 0 and 1 substimulus the same as a density 0 substimulus and a density 5 and 6 substimulus the same as a density 5 one, thereby maintaining the slope. The fact that they did not shows that their decision rules are not based on a simple strategy such as counting number of reinforcers or discriminating simple rates of reinforcement. Instead, they showed the same pattern of anchoring the end

Table 3-2. A Comparison of Obtained Sensitivity to Maximally Possible Sensitivity.

Bird	Six 3-second Cycles		Four 3-second Cycles		Four 4-second Cycles	
	Obtained $d'$	Maximal $d'$	Obtained $d'$	Maximal $d'$	Obtained $d'$	Maximal $d'$
102	1.78	2.99	2.0	2.19	2.17	1.19
84	1.83	2.99	1.8	2.19	1.8	2.19
995	1.70	2.99	1.9	2.19	1.7	2.19
		(E) <sup>a</sup>		(E)		(E)
		.60	.91	.82	.87	.99
		.61	.82	.82	.87	.82
		.57	.87	.87	.77	.77

a. (E) = efficiency.

points of the density distribution to the same extreme perceived values— $p(L) = 0$  for density 0 stimuli and  $p(L) = 1$  for density 6 stimuli. These are the same as the perceived values found for the four-cycle case. Also, their performance is necessarily limited by the discriminability of the densities since they discriminated the same densities better in the four-cycle case. They seem to scale density in both cases in a linear fashion, matching perceived density to relative expected payoff for indicating that density.

### MATCHING AT THE MACRO AND MOLAR LEVEL IS A CONSEQUENCE OF ADDITIVITY AT THE MOLECULAR LEVEL

In this next section, matching will be shown to be derivable from the present micro-molecular model. While the matching law derived here is for the scaling of reinforcement density, Commons and Ducheny (1981) show that a similar model works for a preference situation that reflects response strength. To show how the matching relationship is determined at the micro-molecular level for the scaled density, the terms in the model are put in a form closely akin to traditional matching language. Here a matching relation is between relative response probability (probability and rate are similar measures) and relative payoff for the response indicating the higher density. Relative response probability is just response probability, since  $p(L) + p(R) = 1$ , making  $p(L)/(p(L) + p(R)) = p(L)$ . Relative response probability is then just perceived density. The micro-molecular model showed that each reinforcer makes an equal contribution to relative perceived density when it is exponentially weighted by the decrement in control. The aggregated effect of the total number of reinforcers in a substimulus is the simple sum of the effects of each single reinforcer.

The fact that perceived substimulus density was shown to be the sum of the contributions of each reinforcer on the cycle in which it occurred implies that perceived density is matched to the payoff for indicating the higher density. The proof involves two steps—how molar level stimuli are defined and how linearity implies matching. The contribution of a reinforcer was shown to be an exponentially decaying function of how far the cycle precedes choice.

Perceived reinforcement density at the molar level is for an entire set of substimuli with the same number of reinforcement opportunities. Each reinforcer that contributed to the density of the group was equally likely to fall in any one cycle. Since for each individual substimulus, the effects of a reinforcer on perceived density add, the number of reinforcers occurring in the molar level substimulus class would be additive.

In the derivation of matching at the molar level from the molecular model, the perceived density,  $D''(S_{N_d})$ , of the molar substimulus,  $S_{N_d}$ , is defined in a slightly different manner than previously. Here the perceived density is the average of the perceived density of each same density substimulus,  $z_{p(L|S_n)}$ , already in  $z$  form, rather than the  $z$  transform of the average  $p(L)$ s. The matching relation for this perceived value and payoff for the 2-second standard data is just as good; the  $r^2$ s were all greater than .98 and equal two places beyond the decimal point.

Some of the properties that make this derivation possible depend on empirical properties embedded in the molecular model—specifically, the symmetrical dispersion of  $D^*(0000)$  and  $D^*(1111)$  around 0. This allows the value of the  $\Sigma a_i$ , the sum of the contributions of each reinforcer, to be found in terms of  $a_0$ , the perceived value of (0000).

The following molecular model is assumed to be true:

$$D^*(S_n) = z_{p(L|S_n)}^* = a_0 + \sum_{i=1}^4 v_i a_i$$

Because the value of the contributions of a reinforcer on a given cycle,  $\Sigma a_i$ , will be needed later, it is found next.

$$\text{The value } D^*(0000) = a_0 + \sum_{i=1}^4 0 \cdot a_i = a_0 = -2.16$$

$$\begin{aligned} \text{and the value } D^*(1111) &= a_0 + \sum_{i=1}^4 1 \cdot a_i = a_0 + \sum_{i=1}^4 a_i \\ &= 2.03 \approx 2.16 \end{aligned}$$

Then, by solving the equations simultaneously:

$$\sum_{i=1}^4 a_i = -2a_0$$

The next part of the derivation shows how many  $a_i$ s appear in each molar substimulus,  $S_{N_d}$ . It is necessary to know how many substimuli of the same density there are in each molar substimulus. The cardinality of  $S_{N_d}$  is just the combinations of 4 cycles taken  $N_d$  at a time:

$$\begin{aligned}
 CS_{N_d} &= C \{S_n | D(S_n) = N_d\} = \binom{4}{N_d} = \frac{4!}{N_d!(4-N_d)!} \text{ Then} \\
 D^*(S_{N_d}) &= \frac{1}{\binom{4}{N_d}} \sum_{S_n \in S_{N_d}} D(S_n) \\
 &= \frac{1}{\binom{4}{N_d}} \sum_{S_n \in S_{N_d}} (a_0 + \sum_{i=1}^4 a_i v_i)
 \end{aligned}$$

To expand this expression, the appropriate number of 1s and 0s have to be put in for  $v_i$  for each of the substimuli of a given density. The number of reinforcers ( $v_i = 1$ ) per substimulus is  $N_d$ ; the number of distinct substimuli,  $S_n$ , in the class of same density substimuli,  $S_{N_d}$ , is  $\binom{4}{N_d}$ . Therefore, a total of  $N_d \binom{4}{N_d}$  terms with  $v_i = 1$  occur in the sum. Each of the four cycles contains a reinforcer for an equal fraction of the time, so the total number of times each  $a_i$  is multiplied by a  $v_i = 1$  is  $\frac{1}{4} N_d \binom{4}{N_d}$ . Hence:

$$\begin{aligned}
 D^*(S_{N_d}) &= \frac{1}{\binom{4}{N_d}} \sum_{S_n \in S_{N_d}} (a_0 + \sum_{i=1}^4 a_i v_i) \\
 &= \frac{1}{\binom{4}{N_d}} \sum_{S_n \in S_{N_d}} a_0 + \frac{1}{\binom{4}{N_d}} \sum_{S_n \in S_{N_d}} \sum_{i=1}^4 a_i v_i \\
 &= \frac{\binom{4}{N_d}}{\binom{4}{N_d}} a_0 + \frac{N_d}{4} \frac{\binom{4}{N_d}}{\binom{4}{N_d}} \sum_{i=1}^4 a_i \\
 &= a_0 + \frac{N_d}{4} \sum_{i=1}^4 a_i
 \end{aligned}$$

$$\text{but } \sum_{i=1}^4 a_i \approx -2a_0$$

Therefore,

$$= a_0 - \frac{N_d}{2} a_0 = a_0 \left( 1 - \frac{N_d}{2} \right) = D^* (S_{N_d})$$

This last expression is the same for the relative payoff probability shown earlier, so the derivation is complete.

### WHY HIGHER LEVEL MODELS FAIL

Schedule samples have a number of characteristics. At the macro level, they may differ in overall density. While much may be learned from an examination of the relationship between gross density and response rate or choice, what is controlling about schedules and how that control is exerted may lie at a number of lower levels. First, one would expect the most orderliness at the level that the contingencies constrain the most and at the level where the mechanisms embedded within the contingencies work. Here this took place at the molecular and micro levels. The macro level performance simply reflects the accumulation of all the values from the micro, molecular, and molar levels.

There have been three demonstrations that, in a density discrimination situation, birds match their choices to relative expected payoff for making these choices. In the first (Commons, 1979), relative expected payoff was shown to encompass the effects produced by varying the amount of reinforcement for a correct discrimination of density. Matching was obtained to relative expected payoff for an aggregation of standard and probe trials, with changes in perceived value at the standard lengths compensating for the changes at probe lengths. Matching was also obtained when the number of cycles was increased with the discriminability between schedules held close to constant.

There are a number of implications that follow from matching having been obtained. For the small range investigated here of 0 to 4 and 0 to 6 reinforcers, perceived density was linearly related to actual density, with increments in one producing proportional incre-

ments in the other. The perceived value is reflected by probabilities ranging from very close to 0 ( $z = -2.2$ ) to close to one ( $z = +2.2$ ). The  $z_{p(L)}$  values representing perceived density are almost perfectly correlated with the actual densities ranging from 0 to 4 and 0 to 6. This suggests that perceived density lies on a ratio scale. However, the fact that perceived density is linearly related to actual density at the molar level indicates very little about the perceptual process for two reasons:

1. The effect of changing cycle length does not produce a simple change in perceived density; and
2. The perceived value of different substimuli with the same average density varies a great deal.

One class of molar model states that perceived density simply reflects the total number of reinforcers obtained on a trial, irrespective of when they occurred or their distribution. That momentary increases in cycle length produce decrements in perceived value leads to a rejection of these models, because birds should have been insensitive to the temporal spacing.

Any molar model that states that perceived density reflects average reinforcement rate or average interreinforcement time was also rejected as an overall model. These models were closer to the truth, especially if an interaction of momentary increases in cycle length with standard cycle length is allowed. One reason they were rejected is summarized in Table 3-1, where it is shown that the ratio of slopes of the standard to the obtained perceived value greatly exceeded the predicted value of 2 and 3 for cycle length doubling and tripling, respectively. A second reason for rejecting these molar models was that the contribution of a reinforcer to perceived density decreased as a function of how far before choice that reinforcer occurred, as was also found by Commons (1979). Here, when the probability measure of perceived value was put in  $z$  form and number of cycles away from choice of the critical event was logged, the relation between the two was approximately linear, although there was a good deal of variability for some individual points. A slightly better fit would have been obtained if the exponential transformation used in the micro-molecular model had been used.

To the extent that the micro-molecular model is successful, it suggests the following processes: Perceived density is determined by

decremental weighting, with the number of reinforcers weighted by a negative exponential of their number of cycles before choice. Changes in cycle length did not appear to change the exponent. The effects of the number of events or expected number of events or expected duration of a stimulus period may potentially be described by the coefficient multiplying the exponential terms. Narens (1979) has suggested that varying cycle length from cycle to cycle within a trial, while keeping overall substimulus length constant, may clarify some of these interference and timing issues. Herrnstein (1980) has suggested using fractional probes such as one-half the standard cycle length to see the effect of the probe per se. Including probes that change cycle number but not overall substimulus duration should also be tried.

The results here are entirely consistent with those found by Mandell (Chapter 2). Changing standard cycle length produced minimal changes in discriminability, and the relative time between reinforcers within a substimulus was much more important in determining performance than absolute cycle length in degrading performance. One would predict that discrimination of much leaner densities—that is, much greater cycle lengths—would not be terribly difficult.

### SUMMARY

The way that reinforcers sampled from a schedule are discriminated and scaled is best explained at the micro and molecular level. Each reinforcer as it is weighted makes an equal additive contribution to both the discriminative choice behavior as measured by  $d'$  and the scaled or perceived density as measured by  $z_p(L)$ . The weighted value of a reinforcer was the negative exponential of how many cycles before choice that reinforcer fell. Since cycle length per se did not matter, a decay theory based on number of intervening events was supported. Sensitivity was lower when six 3-second cycle samples were discriminated than when the equivalent number of 4-second cycle samples were discriminated, also supporting a decay theory based on number of intervening events. Furthermore, probes that momentarily increased cycle length (and hence sample length) caused a decrease in perceived density greater than the amount predicted by a model asserting that rate or time between reinforcers was being scaled. This also raises the possibility that, to the bird, the momen-



tarily increased sample length was perceived as an increase in the number of events to be processed, some of which may have fallen outside of the time or number of events window that the bird is prepared for. This supported a notion that number of events embedded in some time base was controlling perceived density and discriminability.

At the molecular level, the perceived substimulus density is simply the sum of the weighted values of the reinforcers occurring within the sample substimulus. This value systematically deviates from matching, with the reinforcers occurring near choice being overscaled and the ones far way being underscaled. However, at the molar level, matching has been shown to be a direct consequence of the micro-molecular level model holding. The scaled density is not only proportional to the actual density but is equal to the probability of reinforcement for indicating that the particular molar substimulus was rich when put into  $z$  form. This in turn implies that at the macro level, the scaled value of the overall schedules was the same as the perceived value of the molar components with density 3 and density 1. The rich stimulus had a mean density of 3 and had a corresponding scaled density equal to the molar density 3 substimulus. Likewise, the lean substimulus had a mean density of 1 and a scaled density equal to a density 1 substimulus. The discriminability degradations were also completely accounted for by the micro level performance. The weight of each reinforcer decreased as the reinforcer occurred further and further before choice. This decrement accounted for the decrease in  $d'$  from the maximum value of an ideal observer. The details of the latter statement are left to a further exposition.

In Commons and Ducheny (1981), it will be shown that a micro-molecular model in a preference situation (where the substimulus effectiveness rather than scaled value is established) will also explain most of the variance. Since contributions of reinforcers on a cycle will also be additive, the matching law relating the strength of responding to the relative amount of reinforcement will also be demonstrated. What is yet to be done is to show how general the micro-molecular model is, as the length and number of cycles are increased to the values usually found in VI schedules.

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